# Quasi-Lagrangian vertical coordinates in ocean modeling – The isopycnic and hybrid-isopycnic option

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- Numerical models can increase our understanding of the oceanic circulation

- Truncation errors are introduced by the discretization of the Navier-Stokes equations.

- One need to quantify the impact of these trunction errors

⇒ Need for calibration of the tool, i.e., the numerical model

- The largest source of truncation error is introduced by the vertical coordinate choice (for a given horizontal resolution)

- Rotating and stratified fluids => dominance of lateral over vertical transport.

- Hence, it is traditional in ocean modeling to orient the two horizontal coordinates orthogonal to the local vertical direction as determined by gravity.

- The choice of the vertical coordinate system is the single most important aspect of an ocean model's design (DYNAMO, DAMÉE-NAB).

- The practical issues of representation and parameterization are often directly linked to the vertical coordinate choice (Griffies et al., 2000).

# Contradictory Considerations in Choosing a Vertical Coordinate

1. THE VERTICAL COORDINATE MUST BE MONOTONIC WITH DEPTH FOR ANY STABLY STRATIFIED DENSITY PROFILE.

2. THE  SOLENOIDAL PRESSURE GRADIENT TERM SHOULD BE ABSENT OR RELATIVELY SMALL COMPARED TO THE NON-SOLENOIDAL PRESSURE GRADIENT TERM WITH AN ACCURATE EQUATION OF STATE

$$\frac{1}{\rho}\nabla_z p = \frac{1}{\rho}\nabla_S p + \nabla_S \phi = \nabla_S\left(\frac{p}{\rho} + \phi\right) + p\nabla_S \frac{1}{\rho}$$

3. MATERIAL CHANGES IN DENSITY DUE TO NUMERICS SHOULD BE MUCH SMALLER THAN CHANGES DUE TO PHYSICAL PROCESSES.

4. COORDINATE SURFACES SHOULD COINCIDE WITH LOCALLY-REFERENCED NEUTRAL SURFACES TO PERMIT A NEARLY TWO-DIMENSIONAL REPRESENTATION OF ADVECTION AND ISONEUTRAL MIXING.

5. IT SHOULD BE POSSIBLE TO CONCENTRATE RESOLUTION WHEREVER IMPORTANT PROCESSES OCCUR, INCUDING BOUNDARY LAYERS AND INTERIOR REGIONS OF LARGE GRADIENTS.

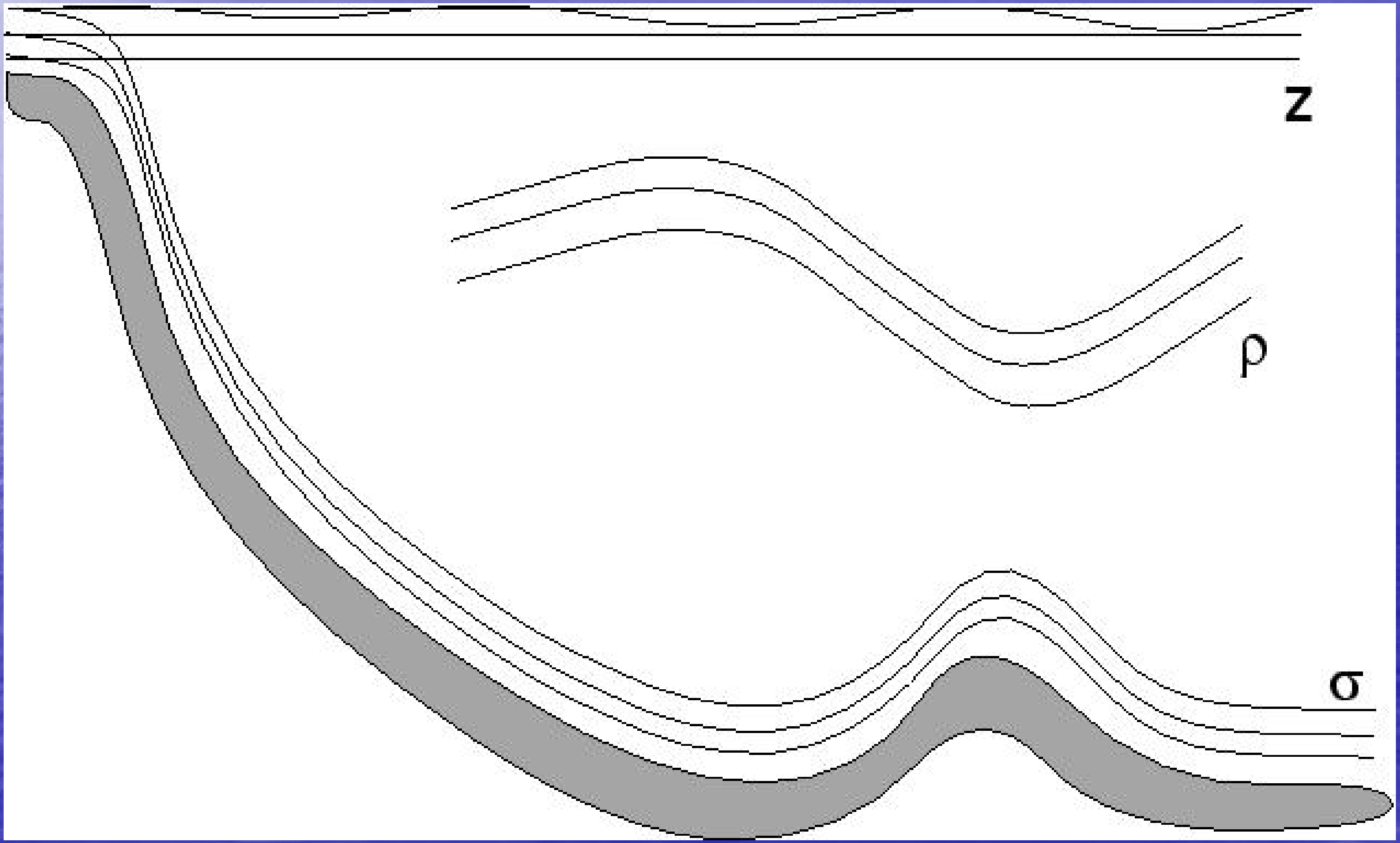
6. CONSISTENCY IS MUCH EASIER TO ESTABLISH WITH A SINGLE VERTICAL COORDINATE

7. THE COORDINATE SHOULD MAKE THE TOP AND BOTTOM BOUNDARY CONDITIONS EASY TO IMPLEMENT EXACTLY.

8. THE COORDINATE SHOULD FACILITATE ANALYSIS OF SIMULATIONS.

Courtesy R. Hallberg, NOAA/GFDL

# Currently, there are **three main vertical coordinates** in use, none of which provides universal utility.



# Brief summary of "Development in Ocean Climate Modeling" by Griffies, Boening, Bryan, Chassignet, Gerdes, Hasumi, Hirst, Treguier, and Webb (2000, Ocean Modelling)

Some key advantages of z-models are:

- The simplest numerical discretization: this has allowed z-models to be used widely soon after their initial development, thus providing valuable years of experience with this class of model.

- For a Boussinesq fluid, the horizontal pressure gradient can be easily represented.

- The equation of state for ocean water can be cleanly and accurately represented.

- The surface mixed layer is naturally parameterized using z-coordinates.

Some disadvantages of z-models are:

- The representation of tracer advection and diffusion along inclined density or neutral surfaces in the ocean interior is cumbersome (spurious diapycnal mixing that can be much larger than the observed background values; see Griffies *et al*. 2000 for details).

- Representation and parameterization of the bottom boundary layer is unnatural.

- Representation of bottom topography is difficult.

Some key advantages of sigma- (or terrain following) models are:

- They provide a smooth representation of the ocean bottom topography, with coordinate isolines concentrated in regions where bottom boundary layer processes are most important. Hence, they allow for a natural framework to parameterize bottom boundary layer processes.

- Thermodynamic effects associated with the equation of state are well represented.

Some disadvantages of sigma-models are:

-The surface mixed layer can be less well represented using sigma than with the z-coordinate. The vertical distance between grid points generally increases upon moving away from the continental shelf regions ⇒ less vertical resolution in the middle of an ocean basin.

- As with the z-models, the representation of advection and diffusion along inclined density surfaces in the ocean interior is cumbersome.

- Sigma-models have difficulty accurately representing the horizontal pressure gradient. The horizontal pressure force consists of two sizable terms, each having separate numerical errors which generally do not cancel ⇒ spurious pressure forces that drive nontrivial unphysical currents.

Some key advantages of isopycnic models are:

- Tracer transport in the ocean interior has a strong tendency to occur along directions defined by locally referenced potential density (i.e., neutral directions), rather than across. Hence, isopycnic models are well suited for representing the dynamics in this regime, so long as isopycnals are reasonably parallel to neutral directions.

- The bottom topography is represented in a piecewise linear fashion, hence avoiding the need to distinguish bottom from side as traditionally done with z-models.

- For an adiabatic fluid, the horizontal pressure gradient can be easily represented.

## Some disadvantages of isopycnic models are

- Representing the effects of a realistic (nonlinear) equation of state is cumbersome, i.e., reference pressure, thermobaricity.

- A density-coordinate is an inappropriate framework for representing the surface mixed layer or bottom boundary layer, since these boundary layers are mostly unstratified.

Main design element of a Lagrangian coordinate: Depth (alias layer thickness or coordinate location) is treated as dependent variable.

⇒ needed: a new independent variable capable of representing 3rd (vertical) model dimension. Call this variable "*s*".

Having increased the number of unknowns by 1 (layer thickness), we need 1 additional equation. The logical choice is an equation linking "*s*" to other variables.

⇒ popular example: *s* = potential density and continuity equation

Hence ….

Main design element of isopycnal models:

Depth and (potential) density trade places as dependent / independent variables

- same number of unknowns, same number of (prognostic) equations, but very different numerical properties

Driving force for isopycnal model development:

**control of numerically induced diapycnal mixing and genetic diversity**

# Continuity equation in generalized ("s") coordinates

$$\begin{pmatrix} \text{vertical} \\ \text{motion} \\ \text{of} \\ s \text{ surface} \end{pmatrix} + \begin{pmatrix} \text{vertical} \\ \text{motion} \\ \text{through} \\ s \text{ surface} \end{pmatrix} = \begin{pmatrix} \text{vertically} \\ \text{integrated} \\ \text{horizontal} \\ \text{mass flux} \\ \text{divergence} \end{pmatrix}$$

**(zero in fixed grids)**

**(zero in material coord.)**

**(known)**

Main benefits:

- explicit PV and potential enstrophy conservation
- reduction of numerically induced diapycnal mixing during advection & diffusion

Main pitfalls:

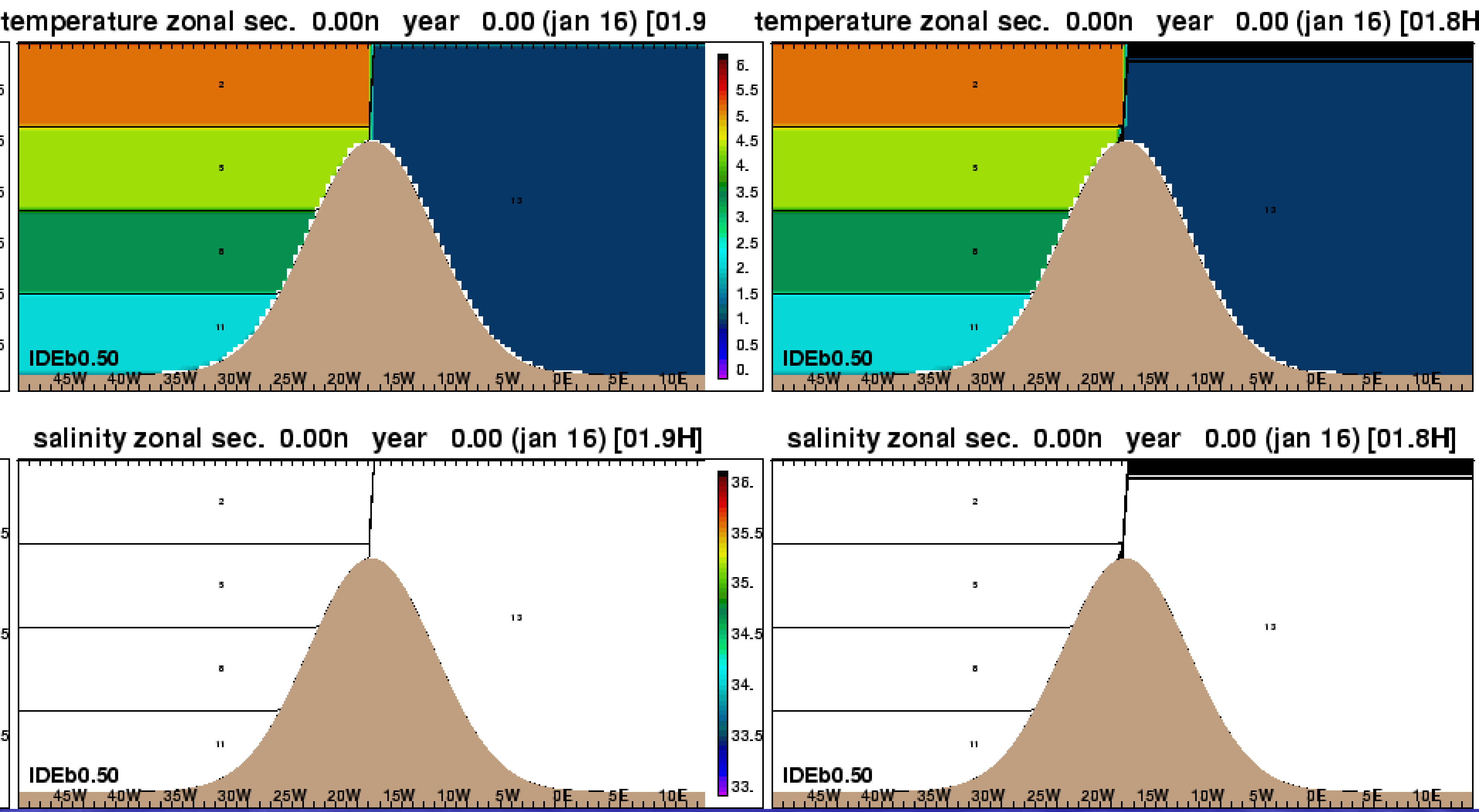
- degeneracy in unstratified water column
- **2**-term horizontal PGF is error-prone in steeply inclined layers (reduction to **1** term possible at the price of approximating state eqn.)
- layer outcropping (=> "massless" layers)
- strongly varying layer thickness requires sophisticated advection schemes

# Overflow Representation

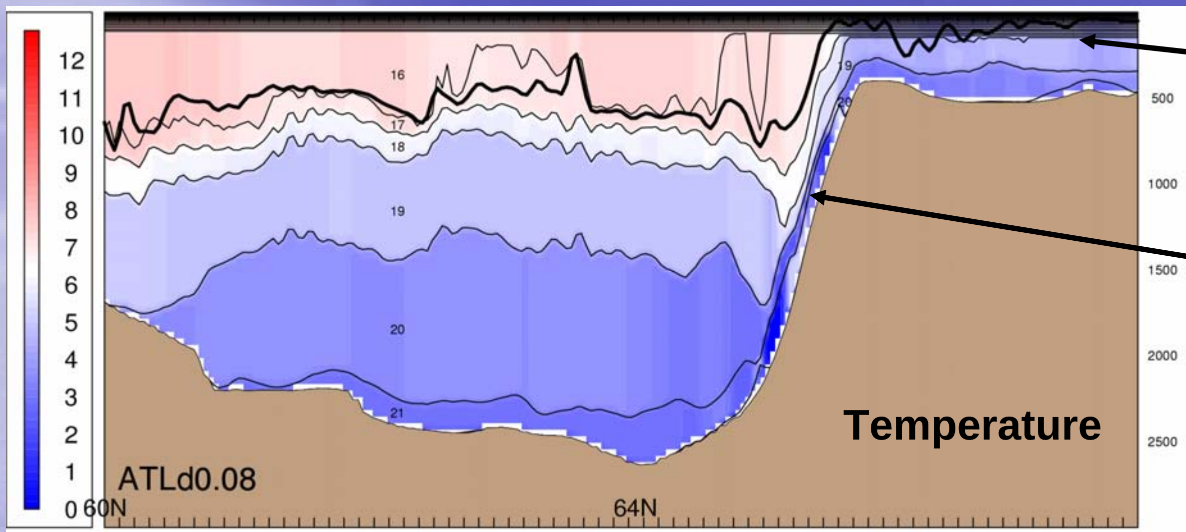
- Strongly dependent on the choice of the vertical coordinate
- In fixed coordinate models (z and σ), the numerically induced entrainment (i.e. mixing) is larger than observed => no need for parameterization, the focus is on reducing the mixing to below observations (lectures by Griffies and Treguier)
- In density coordinate models, the densest fluid will sink to the bottom => need for an entrainment parameterization

# Strong Ri dependence

# Small Ri dependence

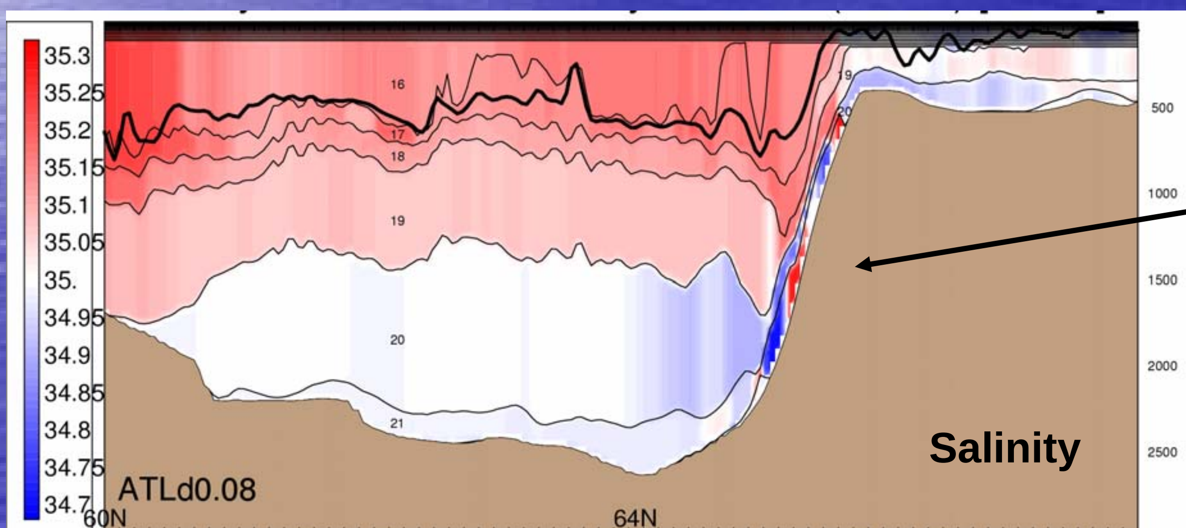


# Denmark Straits Overflow Along 31°W




Colder fresher water forms over the shelf in the Nordic Seas

and spills over the Denmark Strait



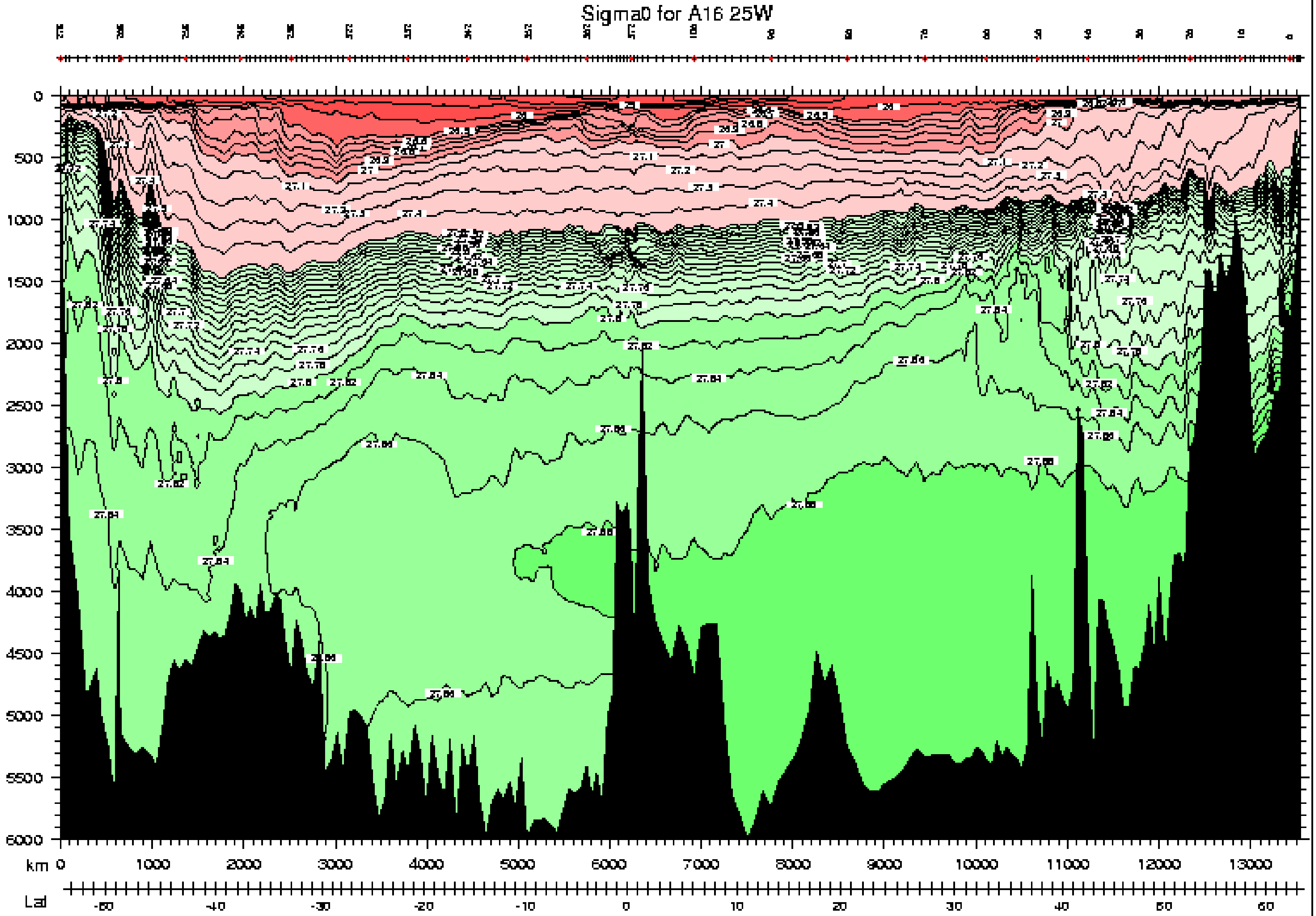

and entrains more saline Irminger Sea water

# Results from 1/12° Atlantic HYCOM

# Vertical coordinate choice in isopycnic coodinate ocean models

**Impact of the coordinate representation and reference density: $\sigma\theta$ , $\sigma 2$ , $\sigma 4$ , $\sigma^* 2$ with correction for thermobaricity**

**(Sun et al., 1999)**


Sigma0 for A16 25W
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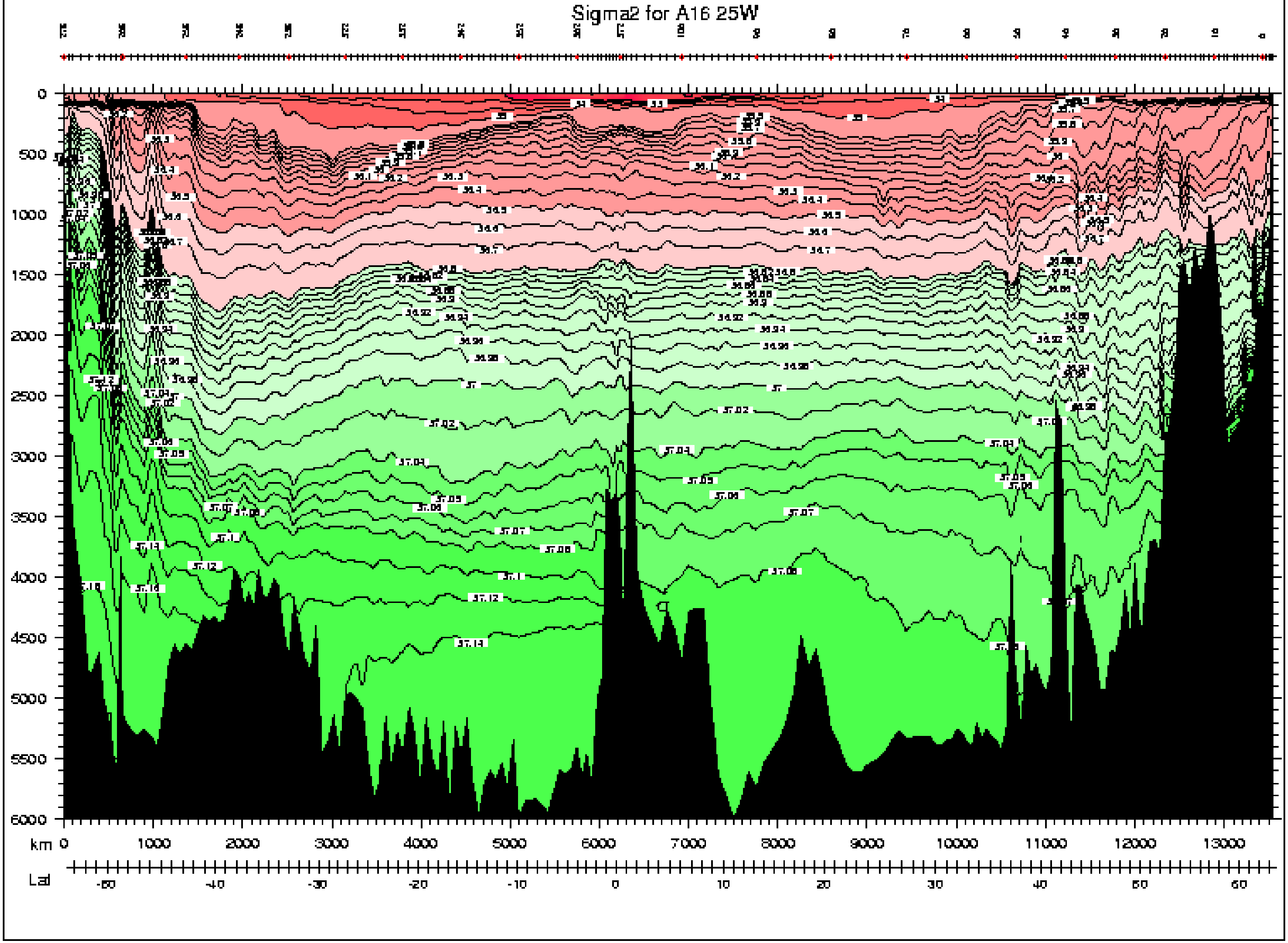

Sigma2 for A16 25W
km
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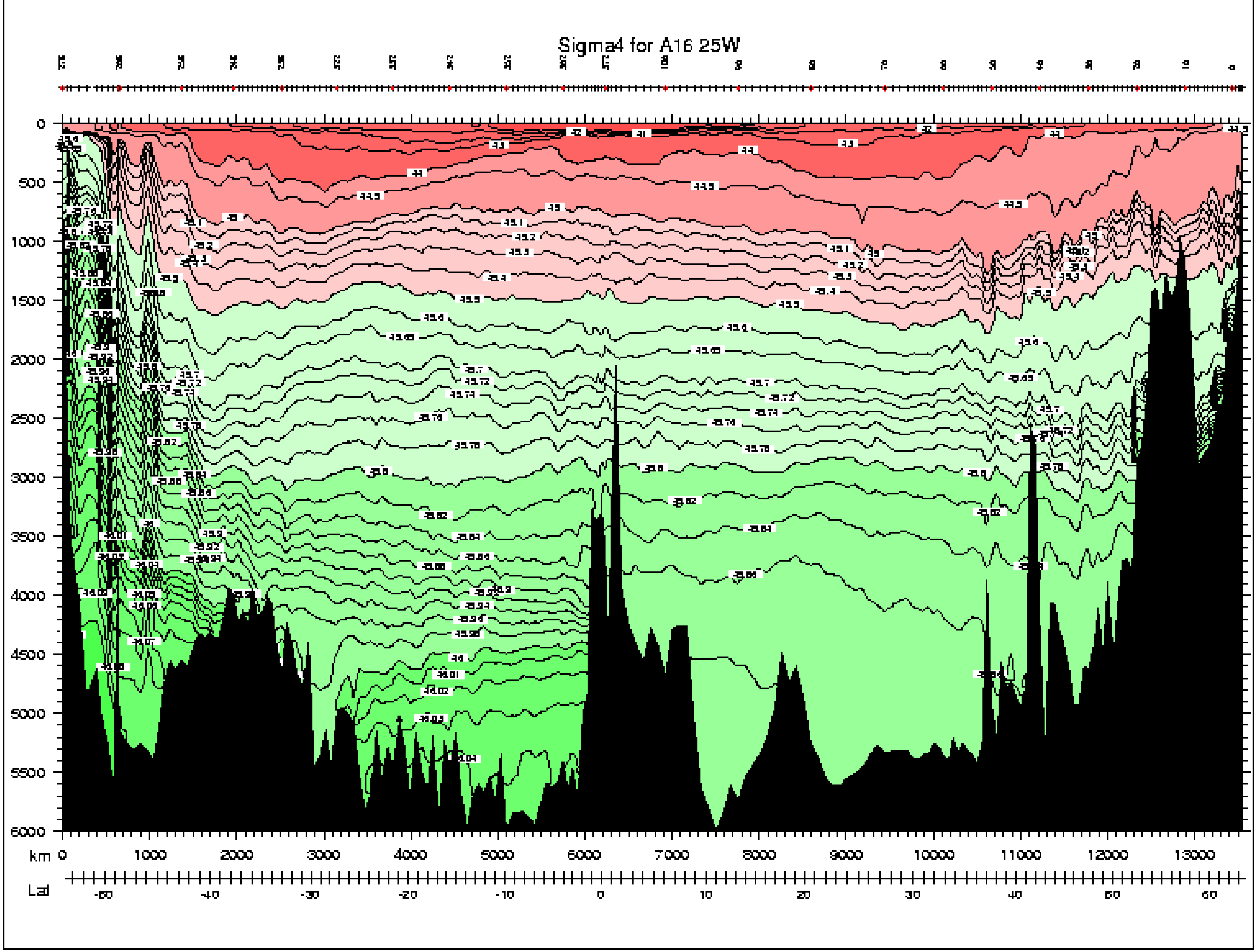
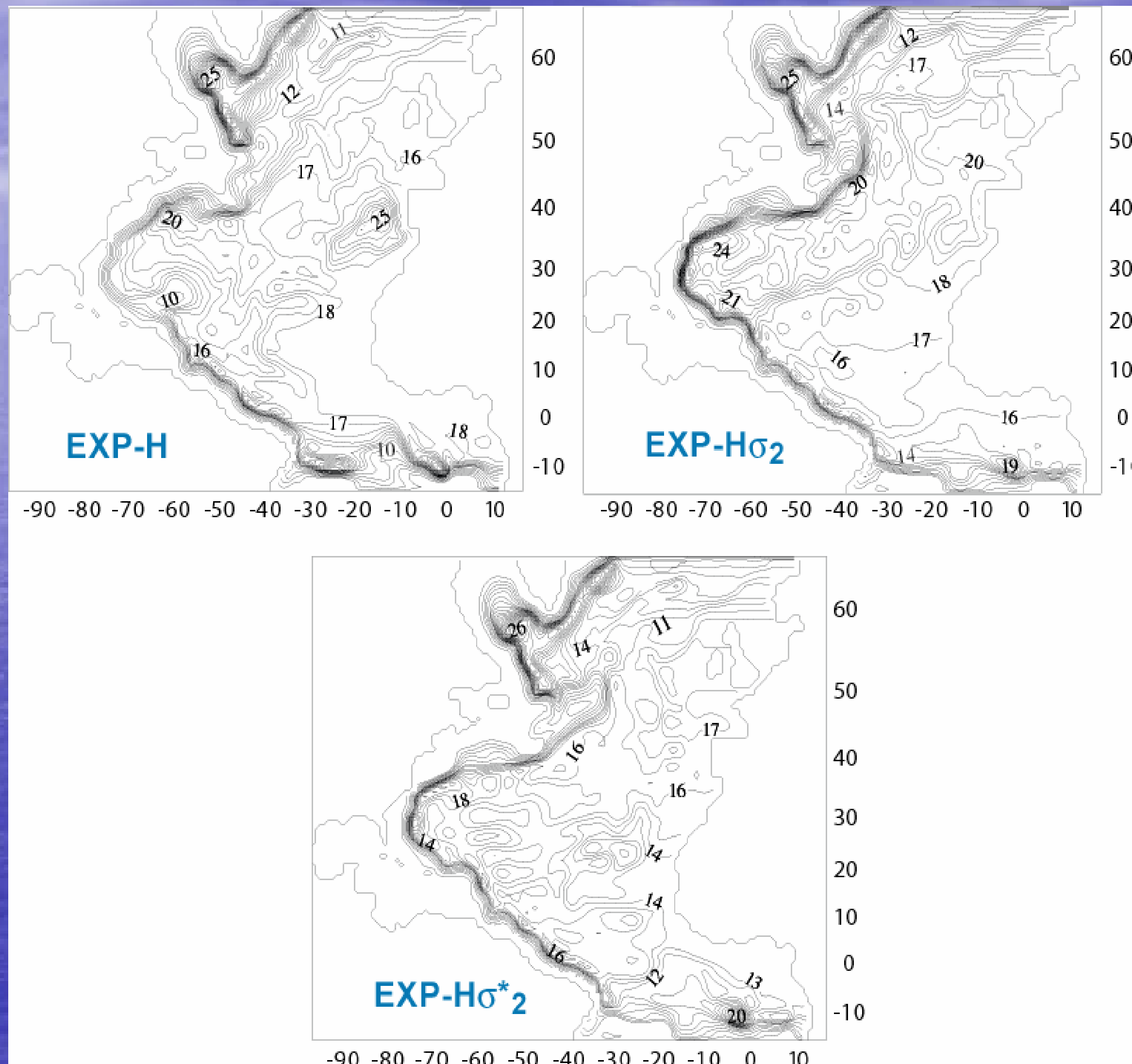

Sigma4 for A16 25W
km
Lat

# Summary of Chassignet et al. (2003, JPO)

- **The main difference** between the $\sigma_\theta$ and $\sigma_2$ experiments is due to the model's representation of AABW since there is no distinct water mass representing AABW in the $\sigma_\theta$ discretization.
- **The differences** between the $\sigma_2$ and $\sigma^*_2$ experiments illustrate the importance of thermobaricity. Without inclusion of the thermobaric effects, the pressure gradient above and below 2000 m does not take into account the modulation of seawater compressibility by potential temperature anomalies. Both the surface and deep circulation are much stronger in the experiment without thermobaricity. It is also only in the $\sigma^*_2$ experiment that the AABW can be seen flowing north along the eastern side of the domain.

# DEEP WATER TRANSPORT




Streamfunction for $\sigma$ > 27.8 (int=1 Sv)

Ideally, an ocean general circulation model (OGCM) should

(a) retain its water mass characteristics for centuries (a characteristic of isopycnic coordinates),

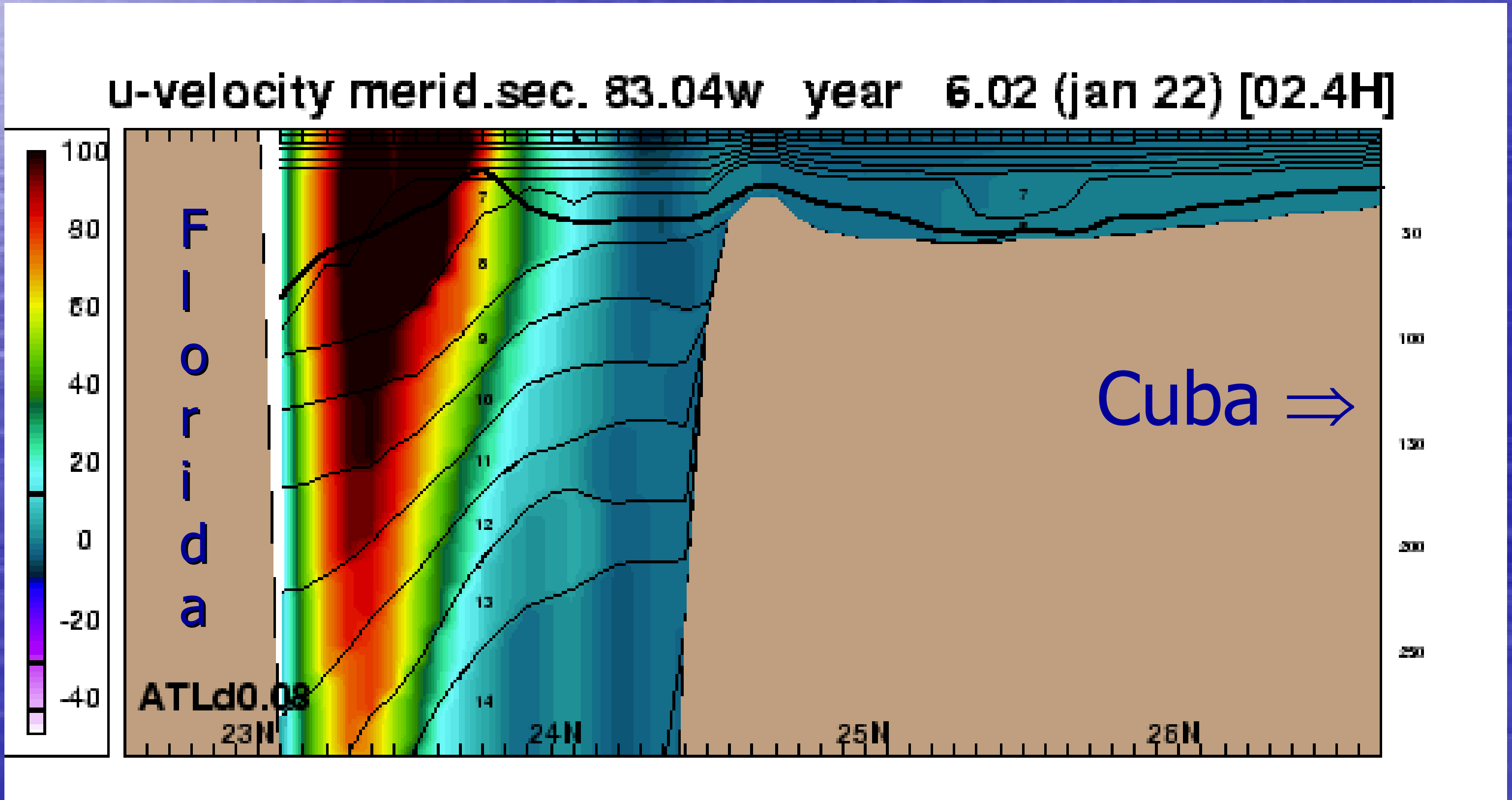
(b) have high vertical resolution in the surface mixed layer (a characteristic of z-level coordinates) for a proper representation of thermodynamical and biochemical processes,

(c) maintain sufficient vertical resolution in unstratified or weakly-stratified regions of the ocean,

(d) have high vertical resolution in coastal regions (a characteristic of terrain-following coordinates).

# HYCOM

**The hybrid coordinate is one that is isopycnal in the open, stratified ocean, but smoothly reverts to a terrain-following coordinate in shallow coastal regions, and to pressure coordinate in the mixed layer and/or unstratified seas.**



# Possible solutions to control numerically induced diapycnal mixing (S. Griffies' lecture)

- Hybrid models with isopycnal interior. Hope they handle the nonlinear equation of state and matching between vertical coordinate regions in a quasi-adiabatic manner.
- Sophisticated numerical advection operators (I have yet to find one that is fully suitable—though still looking)
- Dissipate via quasi-adiabatic operators such as Gent-McWilliams skewsion or variants. This has worked in idealized tests, though research continues. Maybe of use for non-isopycnal models in combination with sophisticated advection operators. Becomes a numerical closure—not a physical closure.

**Grid degeneracy is main reason for introducing hybrid vertical coordinate**

**"Hybrid" means different things to different people:**

**- linear combination of 2 or more conventional coordinates (examples: z+sigma, z+rho, z+rho+sigma)**
**- ALE (Arbitrary Lagrangian-Eulerian) coordinate or generalized coordinate**

ALE maximizes size of isopycnic subdomain.

# ALE: "Arbitrary Lagrangian-Eulerian" coordinate

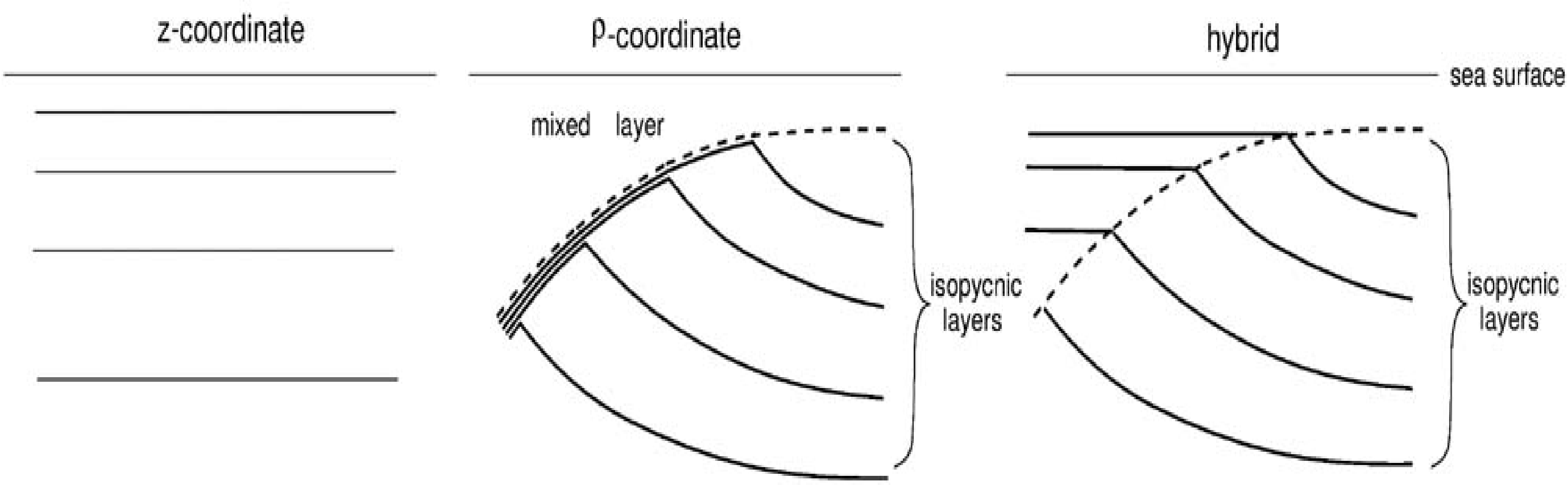
- Original concept (Hirt et al., 1974): maintain Lagrangian character of coordinate but "re-grid" intermittently to keep grid points from fusing.
- In HYCOM (HYbrid Coordinate Ocean Model), we apply ALE in the vertical only and re-grid for 2 reasons:

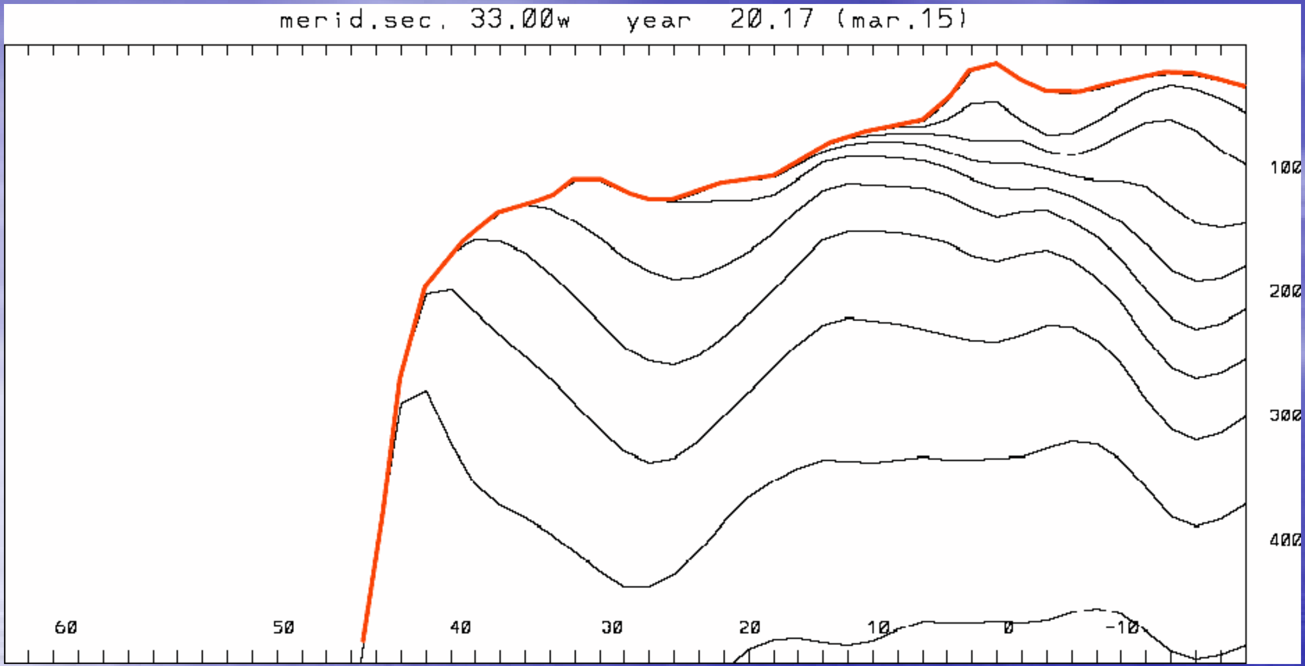
(1) to maintain minimum layer thickness;

(2) to nudge an entropy-related thermo-dynamic variable toward a prescribed layer-specific "target" value by importing water from above or below.

- Process (2) renders the grid **quasi-isentropic**

# Hybrid Coordinates




merid.sec. 33.00w year 20.17 (mar.15)
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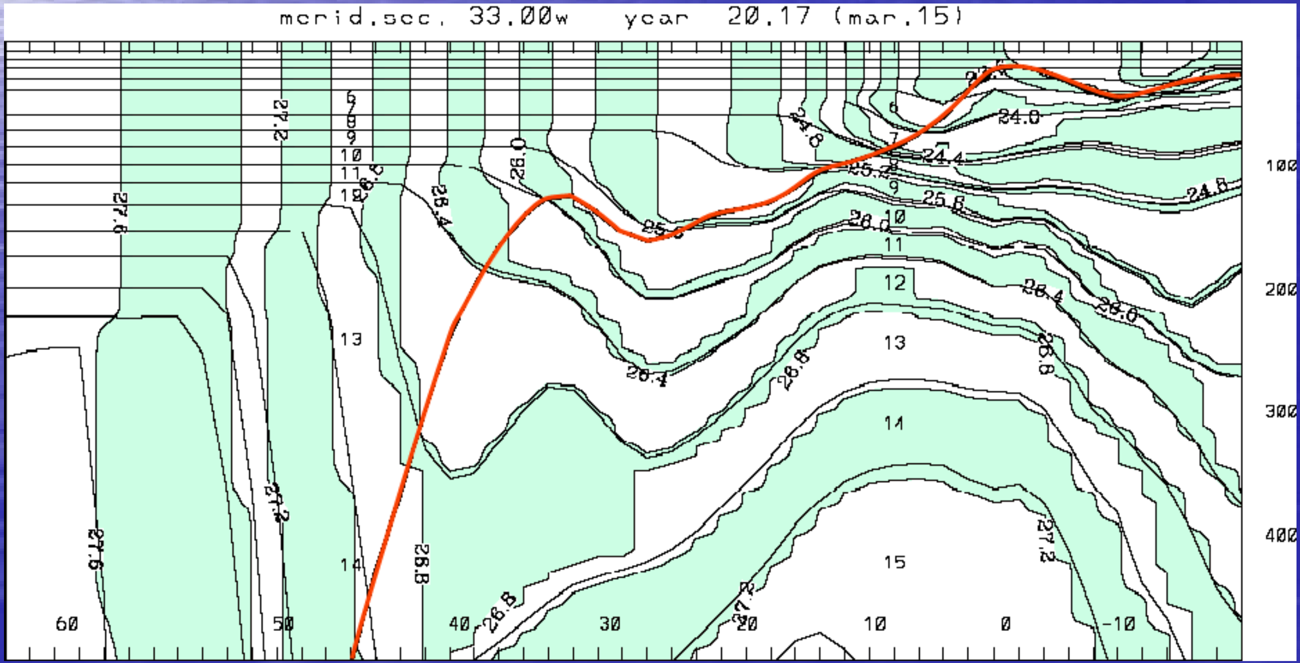
MICOM


merid.sec. 33.00w year 20.17 (mar.15)
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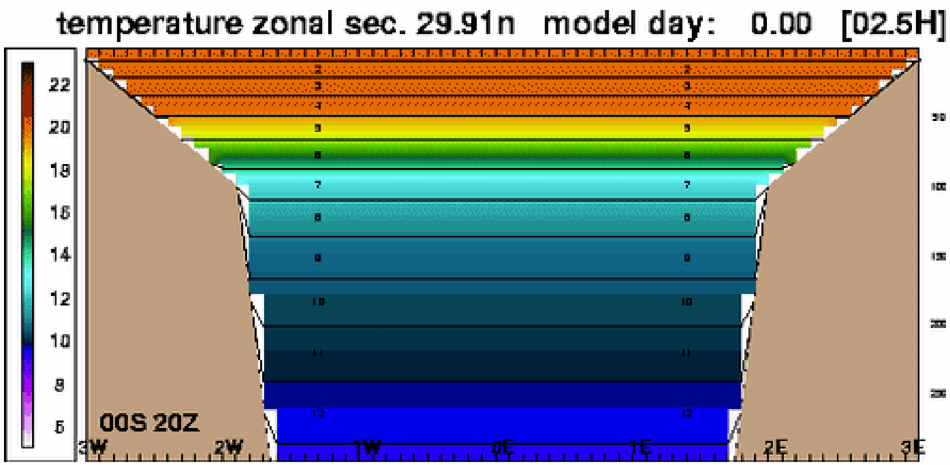
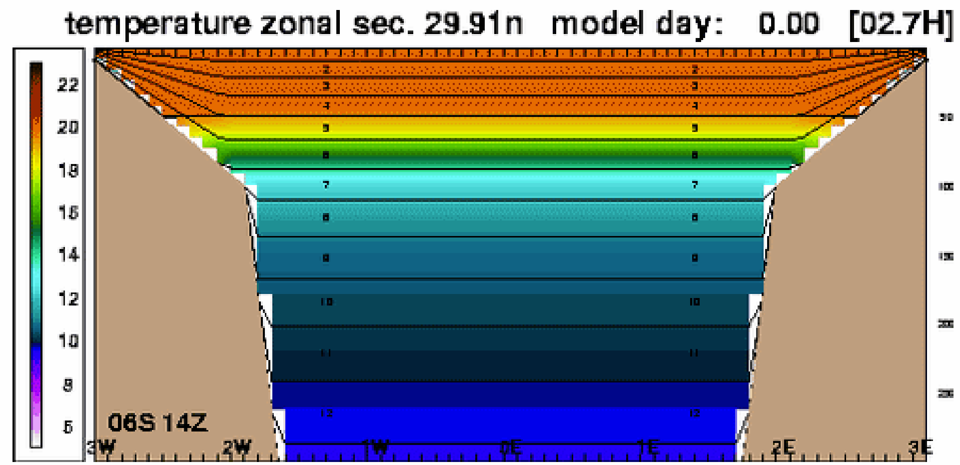
HYCOM

**z** **σ-z**


temperature zonal sec. 29.91n model day: 0.00 [02.5H]
22 20 18 16 14 12 10 8 5
00S 20Z
3W 2W 1W 0E 1E 2E 3E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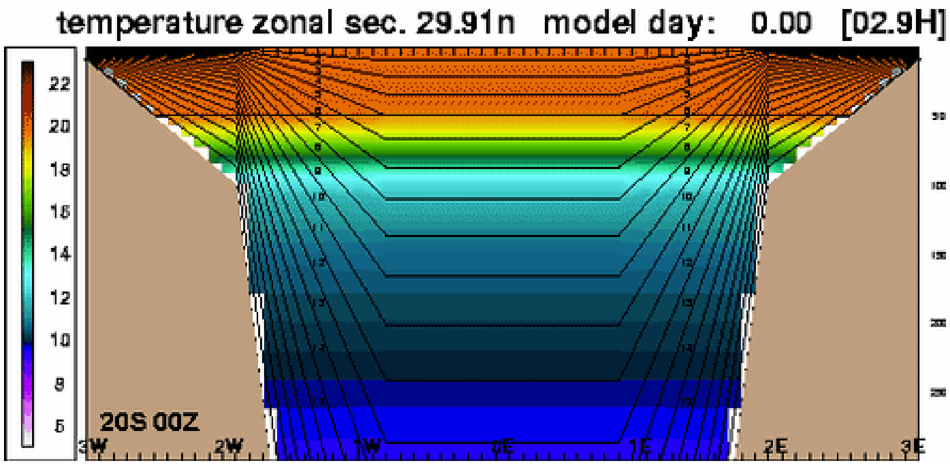
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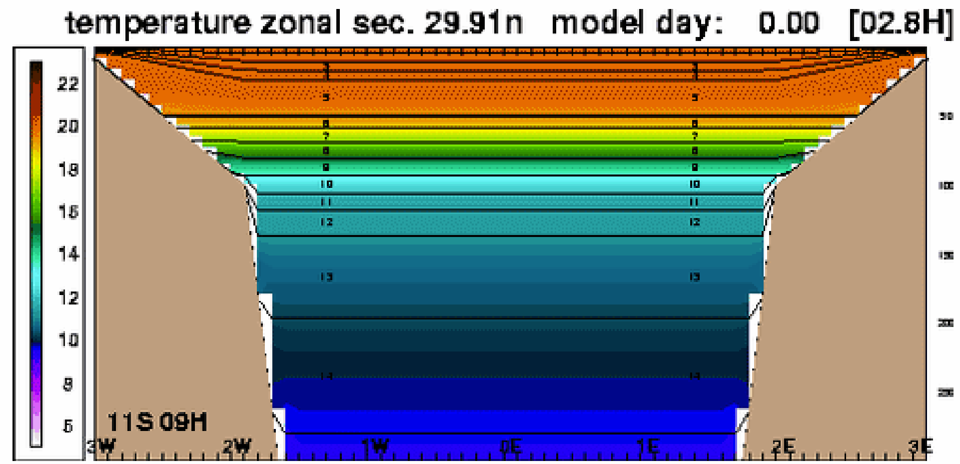

temperature zonal sec. 29.91n model day: 0.00 [02.8H]
22 20 18 16 14 12 10 8 5
11S 09H
3W 2W 1W 0E 1E 2E 3E

**σ** **Hybrid**

# The prototype HYCOM “re-gridder” or “grid generator”

**Design Principles:**

- T/S conservative
- Monotonicity-preserving (no new T/S extrema during re-gridding)
- Layer too dense => entrain lighter water from above
- Layer too light => entrain denser water from below
- Maintain finite layer thickness in upper ocean but allow massless layers on sea floor
- Minimize seasonal vertical migration of coordinate layers by keeping non-isopycnic layers near top of water column.

# The prototype HYCOM grid generator (cont'd)

- Major challenge: achieve smooth **lateral** transition between fixed-depth and isopycnic segments of a coordinate layer.
- Goal: avoid sideways-looking algorithms, i.e., accomplish transition through clever **vertical** re-gridding alone.
- Solution: employ a "cushion" function. Details of the algorithm are as follows ….

# The prototype HYCOM grid generator (cont'd)

- Determine how much water from neighboring layer ("source layer") would be needed to reach target density.
- The amount needed may exceed the amount available in source layer.
- After computing new hypothetical layer thickness $\Delta p$ of source layer (values < 0 allowed), the **cushion function** converts $\Delta p$ into the final layer thickness.

# The prototype HYCOM grid generator (cont'd)

- The **cushion function**, which sets the final thickness of the source layer,
  - leaves large positive $\Delta p$ values unchanged: $cush(\Delta p) = \Delta p$
  - returns a (small) constant value if $\Delta p$ is large negative: $cush(\Delta p) =$ const.
  - links the two cases above by a smoothly varying function for intermediate values of $\Delta p$.

# The prototype HYCOM grid generator (cont'd)

- Negative hypothetical thickness values typically occur under the following conditions:
  - receiving layer is too dense
  - restoration to target density requires more water from source layer than is available.

- The likelihood for this to happen is greatest at high latitudes immediately below the surface

  => high-latitude near-surface layers are more likely to end up with constant thickness than layers elsewhere.

# The prototype HYCOM grid generator (cont.)

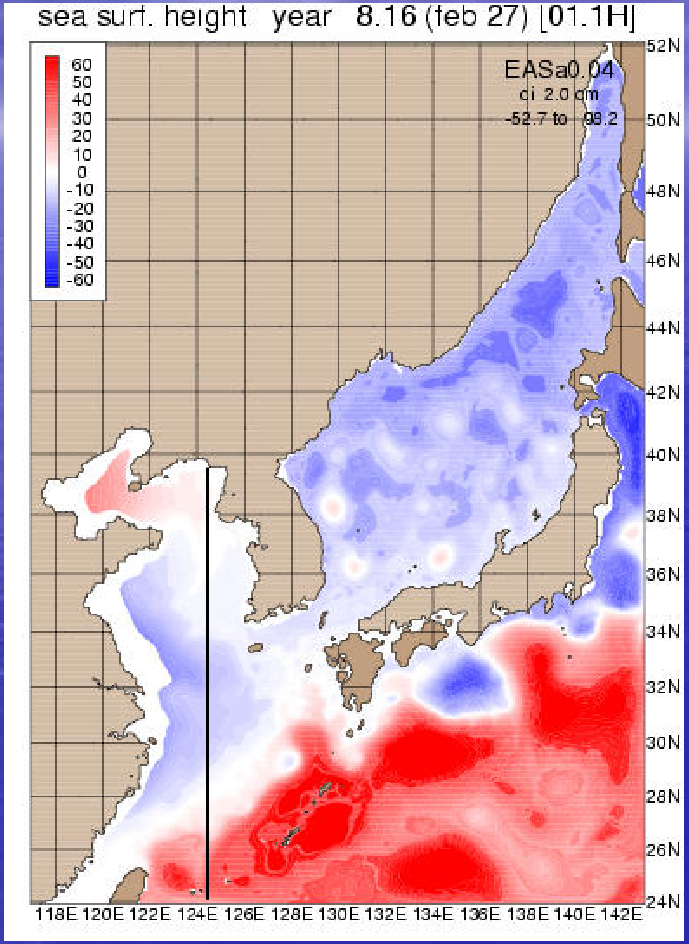
- Example of a cushion function:

$$cush(\Delta p) = \begin{cases} \Delta p & \ldots \; if \quad \dfrac{\Delta p}{\delta} > 3 \\ \delta\left[1 + \left(\dfrac{\Delta p}{3\delta}\right) + \left(\dfrac{\Delta p}{3\delta}\right)^2\right] & \ldots \; if \; -1.5 < \dfrac{\Delta p}{\delta} < 3 \\ 0.75\delta & \ldots \; if \; -1.5 > \dfrac{\Delta p}{\delta} \end{cases}$$

where $\delta$ is a suitably chosen reference thickness. Note continuous first derivative at transition points $\Delta p = -1.5\delta$ and $\Delta p = 3\delta$.

# 1/25° HYCOM East Asian Seas Model

## Nested inside 1/6° HYCOM Pacific Basin Model

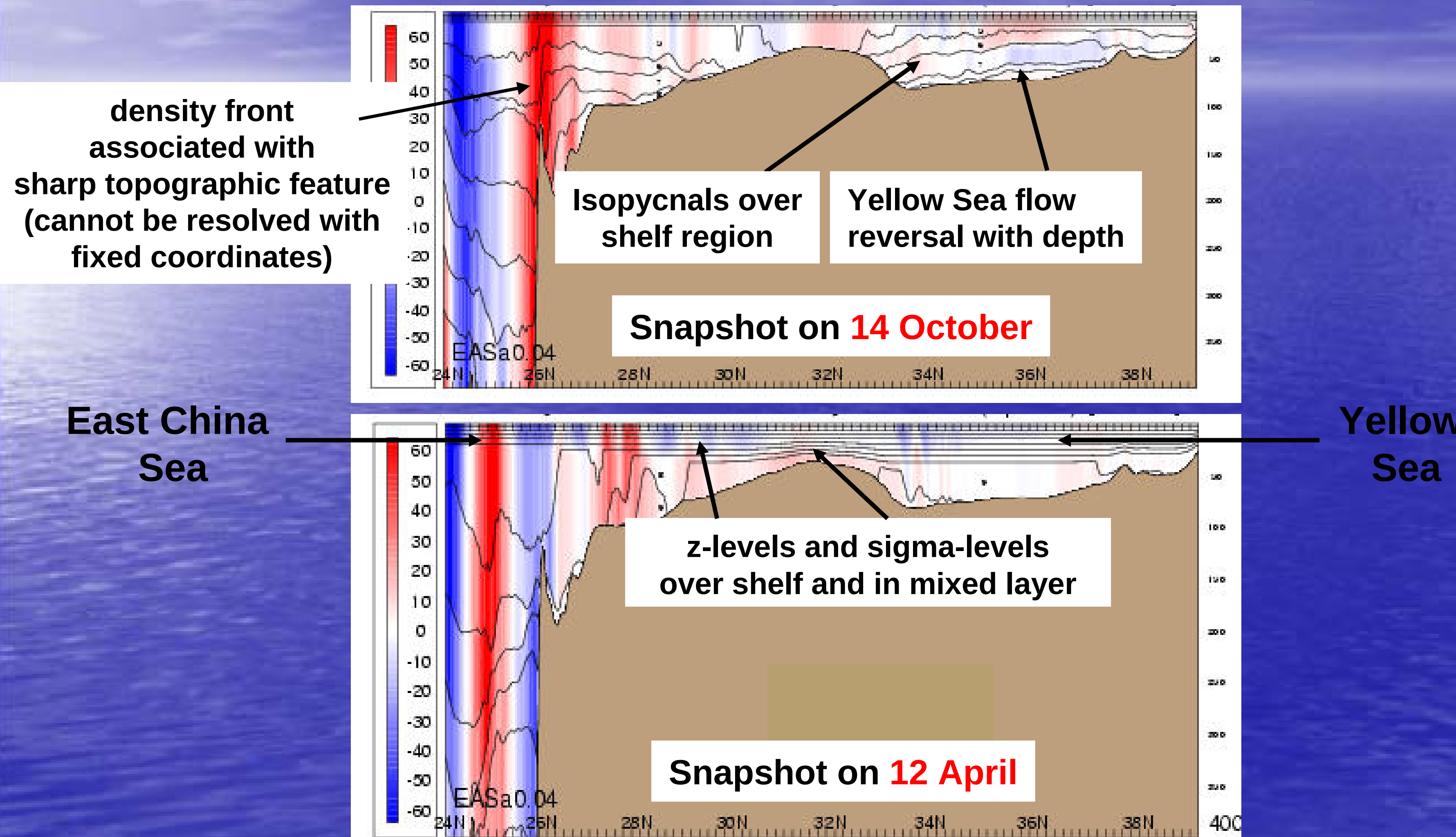



**Boundary conditions via one-way nesting and 6 hrly ECMWF 10 m atmospheric forcing**

# 1/25° East Asian Seas HYCOM
# (nested inside 1/6° Pacific HYCOM)

**North-south velocity cross-section along 124.5°E, upper 400 m**

**blue=westward flow** **red=eastward flow**



# The Hybrid Ocean Modeling Environment (HOME)

A Vision for Community Ocean Circulation Models:
Generalized Vertical Coordinates

- Development: A versatile, open-source, community Ocean Modeling Environment using a generalized hybrid vertical coordinate.
- Science: Study best practices for modeling various important oceanic phenomena.

# The 10-year Vision

- Precursor ocean models disappear.
- Artificial fault lines of ocean modeling community based on vertical coordinate ($\rho$ vs. Z vs. $\sigma$) are erased.
- The same ocean modeling codes usable for education, research, and operations.
- Open, international, and multi-disciplinary.