Chapter 11
Isopycnic and Hybrid Ocean Modeling in the Context of GODAE

Eric P. Chassignet

Abstract An ocean forecasting system has three essential components (observations, data assimilation, numerical model). Observational data, via data assimilation, form the basis of an accurate model forecast; the quality of the ocean forecast will depend primarily on the ability of the ocean numerical model to faithfully represent the ocean physics and dynamics. Even the use of an infinite amount of data to constrain the initial conditions will not necessarily improve the forecast against persistence of a poorly performing ocean numerical model. In this chapter, some of the challenges associated with global ocean modeling are introduced and the current state of numerical models formulated in isopycnic and hybrid vertical coordinates is reviewed within the context of operational global ocean prediction systems.

11.1 Introduction

The main purpose of this chapter is to review the current state of numerical models formulated in isopycnic and hybrid vertical coordinates and discuss their applications within the context of operational global ocean prediction systems and GODAE (Global Ocean Data Assimilation Experiment). In addition to this author's work, this review chapter relies heavily on articles, notes, and review papers by R. Bleck, S. Griffies, A. Adcroft, and R. Hallberg. Appropriate references will be made, but it is inevitable that some similarities in content and style to these publications will be present throughout this chapter.

As stated in Bleck and Chassignet (1994), numerical modeling of geophysical fluid started half a century ago with numerical weather prediction. Ocean model development lagged behind that of atmospheric models, primarily because of

E. P. Chassignet (✉)
Center for Ocean-Atmospheric Predictions Studies, Florida State University,
Tallahassee, FL 32306, USA
e-mail: echassignet@coaps.fsu.edu

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the societal needs for meteorological forecasts, but also because of the inherently greater complexity of circulation systems in closed basins and a nonlinear equation of state for seawater. Furthermore, the computing power required to resolve the relevant physical processes (such as baroclinic instabilities) is far greater for the ocean than the atmosphere since these processes occur on much smaller scale in the ocean. Historically, ocean models have been used primarily to numerically simulate the dominant space-time scales that characterize the ocean system. Simulations of physical integrity require an ability to both accurately represent the various phenomena that are resolved, and to parameterize those scales of variability that are not resolved (Chassignet and Verron 1998). For example, the representation of transport falls into the class of problems addressed by numerical advection schemes, whereas the parameterization of subgrid scale transport is linked to turbulence closure considerations. Although there are often areas of overlap between representation and parameterization, the distinction is useful to make and generally lies at the heart of various model development issues.

Before the Navier-Stokes differential equations can be solved numerically, they must be converted into an algebraic system, a conversion process that entails numerous approximations. Numerical modelers strive to achieve numerical accuracy. Otherwise, the discretization or "truncation" error introduced when approximating differentials by finite differences or Galerkin methods becomes detrimental to the numerical realization. Sources for truncation errors are plentiful, and many of these errors depend strongly on model resolution. Examples include horizontal coordinates (spherical and/or generalized orthogonal), vertical and horizontal grids, time-stepping schemes, representation of the surface and bottom boundary layers, bottom topography representation, equation of state, tracer and momentum transport, subgrid scale processes, viscosity, and diffusivity. Numerical models have improved over the years not only because of better physical understanding, but also because modern computers permit a more faithful representation of the differential equations by their algebraic analogs.

A key characteristic of rotating and stratified fluids, such as the ocean, is the dominance of lateral over vertical transport. Hence, it is traditional in ocean modeling to orient the two horizontal coordinates orthogonal to the local vertical direction as determined by gravity. The more difficult choice is how to specify the vertical coordinate. Indeed, as noted by various ocean modeling studies such as DYNAMO (Meincke et al. 2001; Willebrand et al. 2001) and DAMEE-NAB (Chassignet and Malanotte-Rizzoli 2000), the choice of a vertical coordinate system is the single most important aspect of an ocean model's design. The practical issues of representation and parameterization are often directly linked to the vertical coordinate choice. Currently, there are three main vertical coordinates in use, none of which provide universal utility. Hence, many developers have been motivated to pursue research into hybrid approaches.

As outlined by Griffies et al. (2000a), there are three regimes of the ocean that need to be considered when choosing an appropriate vertical coordinate. First, there is the surface mixed layer. This region is generally turbulent and dominated by transfers of momentum, heat, freshwater, and tracers. It is typically very well mixed in the vertical through three-dimensional convective/turbulent processes. These processes involve non-hydrostatic physics, which requires very high horizontal and vertical resolution to be explicitly represented (i.e., a vertical to horizontal grid aspect ratio near unity). A parameterization of these processes is therefore necessary in primitive equation ocean models. In contrast, tracer transport processes in the ocean interior predominantly occur along constant density directions (more precisely, along neutral directions). Therefore, water mass properties in the interior tend to be preserved over large space and time scales (e.g., basin and decadal scales). Finally, there are several regions where density driven currents (overflows) and turbulent bottom boundary layer processes act as a strong determinant of water mass characteristics. Many such processes are crucial for the formation of deep water properties in the world ocean.

The simplest choice of vertical coordinate (Fig. 11.1) is z, which represents the vertical distance from a resting ocean surface. Another choice for vertical coordinate is the potential density referenced to a given pressure. In a stably stratified adiabatic ocean, potential density is materially conserved and defines a monotonic layering of the ocean fluid. A third choice is the terrain-following σ coordinate. The depth or z coordinate provides the simplest and most established framework for ocean modeling. It is especially well-suited for situations with strong vertical/diapycnal mixing and/or low stratification, but has difficulty in accurately representing the ocean interior and bottom. The density coordinate, on the other hand, is well-suited to modeling the observed tendency for tracer transport to be along density (neutral) directions, but is inappropriate in unstratified regions. The σ coordinate provides a suitable framework in situations where capturing the dynamical and/or boundary layer effect associated with topography is important. Terrain-following σ coordinates are particularly well suited for modeling flows over the continental shelf, but remain unproven in a global modeling context. They have been used extensively for

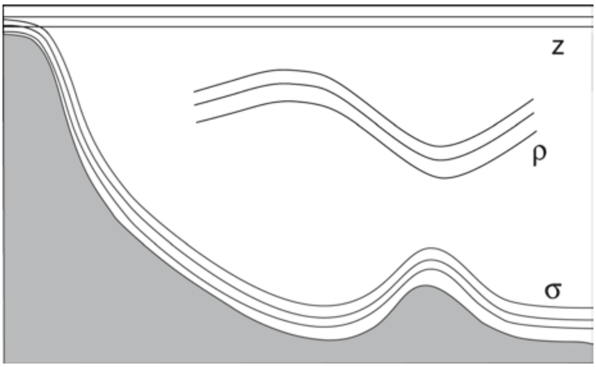

Fig. 11.1 Schematic of an ocean basin illustrating the three regimes of the ocean germane to the considerations of an appropriate vertical coordinate. The surface mixed layer is naturally represented using fixed depth z (or pressure p) coordinates, the interior is naturally represented using isopycnic ρ_{pot} (potential density tracking) coordinates, and the bottom boundary is naturally represented using terrain-following σ coordinates. (Adapted from Griffies et al. 2000a)

coastal engineering applications and prediction (see Greatbatch and Mellor (1999) for a review), as well as for regional and basin-wide studies.

Ideally, an ocean model should retain its water mass characteristics for centuries of integration (a characteristic of density coordinates), have high vertical resolution in the surface mixed layer for proper representation of thermodynamical and biochemical processes (a characteristic of z coordinates), maintain sufficient vertical resolution in unstratified or weakly stratified regions of the ocean, and have high vertical resolution in coastal regions (a characteristic of terrain-following σ coordinates). This has led to the recent development of several hybrid vertical coordinate numerical models that combine the advantages of the different types of vertical coordinates in optimally simulating coastal and open-ocean circulation features.

Within the GODAE context, the global ocean models presently used or tested for ocean forecasting systems can be divided into two categories: fixed coordinates (MOM, NEMO, MITgcm, NCOM, POP, OCCAM, …) or primarily Lagrangian coordinates (NLOM, MICOM, HYCOM, POSEIDON, GOLD, …). The reader is referred to the Appendix for a definition of the acronyms and references.

11.2 Ocean Model Requirements for GODAE

The specific objectives of GODAE are to:

a) Apply state-of-the art ocean models and assimilation methods to produce short-range open-ocean forecasts, boundary conditions to extend predictability of coastal and regional subsystems, and initial conditions for climate forecast models.
b) Provide global ocean analyses for developing improved understanding of the oceans and improved assessments of the predictability of ocean variability, and for serving as a basis for improving the design and effectiveness of a global ocean observing system.

The requirements for the ocean model differ among these objectives. High-resolution operational oceanography requires accurate depiction of mesoscale features, such as eddies and meandering fronts and of upper ocean structure. Coastal applications require accurate sea level forced by wind, tidal forces, and surface pressure. Seasonal-to-interannual forecasts require a good representation of the upper ocean mass field and coupling to an atmosphere. This diversity of applications implies that no single model configuration will be sufficiently flexible to satisfy all the objectives.

For high-resolution operational oceanography (see Hurlburt et al. (2008, 2009) for a review), the models have to be global and eddy-resolving, with high vertical resolution and advanced upper-ocean physics, and use high-performance numerical code and algorithms. To have a good representation of the mesoscale variability, the horizontal grid spacing must be fine enough to be able to resolve baroclinic instability processes. Most numerical simulations to date suggest that a minimum grid spacing on the order of 1/10° (see AGU monograph by Hecht and Hasumi 2008 for a review) is needed for a good representation of western boundary currents (including their separation from the coast) and of the eddy kinetic energy. The computational requirements for global ocean modeling at this resolution are extreme and demand the latest in high-performance computing. For that reason, there are only a few eddy-resolving global ocean models currently being integrated with or without data assimilation: NLOM 1/32° (Shriver et al. 2007), POP 1/10° (Maltrud and McClean 2005), HYCOM 1/12° (Chassignet et al. 2009), and MERCATOR/NEMO 1/12° (Bourdallé-Badie and Drillet personal communication).

11.3 Challenges

As the mesh is refined, ocean models faceseveral challenges. This section summarizes the challenges that this author thinks are most relevant to GODAE's goal of high-resolution operational oceanography.

Model-Related Data Assimilation Issues In data assimilation, there is a much larger burden on ocean models than on atmospheric models because (1) synoptic oceanic data is overwhelmingly at the surface, (2) ocean models must use simulation skills in converting atmospheric forcing into an oceanic response, and (3) ocean model forecast skill is needed in the dynamical interpolation of satellite altimeter data (since the average age of the most recent altimeter data on the repeat tracks is 1/2 the repeat cycle plus the delay in receiving the real-time data, typically 1–3 days at present). Specifically, the model must be able to accurately represent ocean features and fields that are inadequately observed or constrained by ocean data. This is an issue for re-analyses, for real-time mesoscale resolving nowcasts and short-range forecasts (up to ~1 month), and for seasonal-to-interannual forecasts, including the geographical distribution of anomalies. Ocean simulation skill is especially important for mean currents and their transports (including flow through straits), the surface mixed layer depth, Ekman surface currents, the coastal ocean circulation, the Arctic circulation, and the deep circulation (including the components driven by eddies, the thermohaline circulation, and the wind).

To order to assimilate the SSH (Sea Surface Height) anomalies determined from satellite altimeter data into the numerical model, it is necessary to know the oceanic mean SSH over the time period of the altimeter observations. Unfortunately, the earth's geoid is not presently known with sufficient accuracy to provide an accurate mean SSH on scales important for the mesoscale. Several satellite missions are underway or planned to help determine a more accurate geoid, but not on a fine enough scale to entirely meet the needs of mesoscale prediction. Thus, it is of the utmost importance to have a model mean that is reasonably accurate since most oceanic fronts and mean ocean current pathways cannot be sharply defined from hydrographic climatologies alone.

A number of additional issues, theoretical or technical, are raised when the numerical ocean model is used in conjunction with data assimilation techniques. In all data assimilation methods, nonlinearities are a major source of sub-optimality. Variational methods often require development of the adjoint model, which is a demanding task. Depending on the vertical coordinates, difficulties arise in dealing with non-Gaussian statistics in isopycnic coordinate models with vanishing layers, or with convective instability processes throughout the vertical columns in z coordinate models. Finally, defining prior guess errors, model errors, and, to a lesser degree, observation errors, is difficult.

Forcing The ocean model will respond to the prescribed atmospheric forcing fields. The present models' inability to reproduce the present-day ocean circulation when run in free mode is a consequence of inaccuracies in both the forcing and in the numerical models themselves, as well as of the intrinsic nonlinearity of the Navier-Stokes equations. Accurate atmospheric forcing, when computed using bulk formulas that combine the model SST and the atmospheric data, have been shown to be essential for a successful forecast of the sea surface temperature, sea surface salinity, and mixed layer depths. It is important to mention here that the prescription of the surface forcing fields, as currently done in many ocean forecasting systems, does not allow for atmospheric feedback. This may have a limited impact on a 15-day forecast, but coupling to an atmospheric model is essential in seasonal-to-interannual forecasting of events such as ENSO (Philander 1990; Clarke 2008).

Topography With high-resolution modeling comes the need for high-resolution topography. The most commonly used global bathymetric database is the Smith and Sandwell (1997, 2004) database, which is derived from a combination of satellite altimeter data and shipboard soundings. The latest version (http://topex.ucsd.edu/WWW_html/srtm30_plus.html) is at 1/2 min resolution and covers the entire globe, with patches from the IBCAO topography (Jakobsson et al. 2000) in the Arctic and from various high-resolution sounding data where such data are available. Most, but not all, of the other available global bathymetric data sets, for instance, the latest GEBCO bathymetry, ETOPO2, DBDB2, and so on, utilize the Smith and Sandwell database in the deep ocean. Differences can often be found among various bathymetry products in shallow water, where satellite altimetry is much less useful, and where local high-quality datasets are often used. While modern acoustic sounding data can achieve lateral resolutions of about 100 m, such data cover only a small fraction of the open ocean. In areas not covered by such data, the true feature resolution of the Smith and Sandwell datasets is approximately given by π times the water column depth, i.e., about 10–20 km. Goff and Arbic (2010) have recently created a synthetic data set in which the topographic anomalies depend on local geophysical conditions such as seafloor spreading rate. The synthetic topography can be overlaid on the Smith and Sandwell datasets to create global bathymetries that have the right statistical texture (roughness), even if the "bumps" are not deterministically correct.

Meridional Overturning Circulation A good representation of the overturning circulation is essential for a proper representation of the oceanic surface fields. This is especially true in the North Atlantic where the contribution of the thermohaline meridional overturning circulation accounts for a significant portion of the Gulf Stream transport. Many factors, such as mixed layer physics, ice formation, overflow representation, and interior diapycnal mixing, affect the strength and pathways of the meridional overturning circulation.

Ice Models A global ocean model needs to be coupled to an ice model to have the proper forcing at high latitudes and hence the correct dense water mass formation and circulation. A good representation of the ice cycle is challenging, especially when the atmospheric fields are prescribed. Another related issue is the mixed layer parameterization below the ice.

Overflows Sill overflows typically involve passages through the ridge and are under the control of hydraulic effects, each of which is highly dependent on topographic details. The downslope flow of dense water, typically in thin turbulent layers near the bottom, may strongly entrain ambient waters and is modulated by mesoscale eddies generated near the sill. The simulation of downslope flows of dense water differs strongly among ocean models based on different vertical coordinate schemes. In *z* coordinate models, difficulties arise from the stepwise discretization of topography, which tends to produce gravitationally unstable water parcels that rapidly mix with the ambient fluid as they flow down the slope. The result is a strong numerically induced mixing of the outflow water downstream of the sill. This numerically induced mixing will in principle decrease as the horizontal and vertical grid spacing is refined. It is, however, still an issue at the above mentioned resolution of 1/10° (see review article by Legg et al. 2009).

Diapycnal Mixing This observational field is the least well known and the most difficult to model correctly, especially in fixed coordinate models (Griffies et al. 2000b; Lee et al. 2002) due to the typically small levels of mixing in the ocean interior away from boundaries (Ledwell et al. 1993). Excessive numerically induced diapycnal mixing will lead to incorrect water mass pathways and a poor representation of the thermohaline circulation.

Internal Gravity Waves/Tides Improperly resolved internal gravity waves generate numerically induced diapycnal mixing in fixed-coordinate models. Several numerical techniques can be used to slow the gravity waves, but ultimately it would be desirable to have a diapycnal mixing parameterization based on the model representation of internal gravity waves. The inclusion of astronomical tidal forcing in ocean models generates barotropic tides, which in turn generate internal tides in areas of rough topography. Until recently, global modeling of the oceanic general circulation and of tides have been separate endeavors. A first attempt to model the global general circulation and tides simultaneously and at high horizontal resolution is described in Arbic et al. (2010). In contrast to earlier models of the global internal tides, which included only tidal forcing and which utilized a horizontally varying stratification, the stratification can vary horizontally in a model that also includes wind- and buoyancy-forcing. Arbic et al. (2010) show that the horizontally varying stratification affects tides to first order, especially in polar regions. Inclusion of tides in general circulation models is also more likely to properly account for the effects of the quadratic bottom boundary layer drag term. Many ocean general circulation models insert an assumed tidal background flow, typically taken to be about 5 cm/s, into the quadratic drag formulation (e.g., Willebrand et al. 2001). However, in the actual ocean tidal velocities vary from about 1–2 cm/s in the abyss to about 0.5–1 m/s in areas of large coastal tides. Thus an assumed tidal background flow of 5 cm/s is too strong in the abyss and too weak in coastal areas. By actually resolving the (spatially inhomogeneous) tidal flows in a general circulation model, this problem can be corrected.

Barotropic Motions The use of high-frequency (e.g., 3-hourly) forcing generates strong non-steric barotropic motions that are not temporally resolved by satellite altimeters (Stammer et al. 2000). In addition, Shriver and Hurlburt (2000) report that between 5 and 10 cm rms SSH non-steric variability are generated in major current systems throughout the world ocean.

Viscosity Closure Despite the smaller mesh size, the viscosity parameterization remains of importance for the modeled large-scale ocean circulation (Chassignet and Garraffo 2001; Chassignet and Marshall 2008; Hecht et al. 2008). When the grid spacing reaches a certain threshold, the energy cascade from small to large scales should be properly represented by the model physics. Dissipation should then be prescribed for numerical reasons only to remove the inevitable accumulation of enstrophy on the grid scale. This is the reason higher-order operators such as the biharmonic form of friction have traditionally been favored in eddy-resolving or eddy-permitting numerical simulations. Higher-order operators remove numerical noise on the grid scale and leave the larger scales mostly untouched by allowing dynamics at the resolved scales of motion to dominate the subgrid-scale parameterization (Griffies and Hallberg 2000). In addition to numerical closure, the viscosity operator can also be a parameterization of smaller scales. One of the most difficult tasks in defining the parameterization is the specification of the Reynolds stresses in terms of only the resolved scales' velocities. The common practice has been to assume that the turbulent motion acts on the large-scale flow in a manner similar to molecular viscosity. However, the resulting Laplacian form of dissipation removes both kinetic energy and enstrophy over a broad range of spatial scales, and its use in numerical models in general implies less energetic flow fields than in cases with more highly scale-selective dissipation operators. Some Laplacian dissipation is still needed to define viscous boundary layers and to remove eddies on space scales too large to be removed by biharmonic dissipation and too small to be numerically accurate at the model grid resolution.

Coastal Transition Zones A strong demand for ocean forecasts will come from the offshore industry, which has extended its activities from the shallow shelf seas to exploration and production on the continental slope, where oceanographic conditions play a much more critical role in safe and environmentally acceptable operations. Exploration and production are now taking place in water depths in excess of 2000 m in a number of oil and gas basins around the world. The proper modeling of the transition area between the deep ocean and the shallow continental shelves imposes strong requirements on the ocean model. It should be capable of modeling the typical shallow waters on the shelf, with their characteristic well-mixed water masses and strong tidal and wind-driven currents. Furthermore, it also must properly represent and distinguish between water masses of vastly different characteristics in the deep ocean and near the surface during very long time integrations. The interaction with the continental shelf/slope is also an intriguing problem due to the impact on internal tides and the wave modes developing and propagating along the continental shelf/slope. This includes remotely generated wave modes, such as equatorially generated Kelvin waves, which play a large role in El Niño events and which can strongly impact distant coastal regions.

11.4 On the Use of Potential Density as a Vertical Coordinate

As stated in the introduction, the choice of a vertical coordinate system is the single most important aspect of an ocean model's design and, because of the practical issues of representation and parameterization, many of the challenges listed in the previous sections are directly linked to the vertical coordinate choice. There is no "best" choice of vertical coordinate since all solutions of the discretized equations with any vertical coordinate should converge toward the solutions of the corresponding differential equations as mesh size goes to zero. Each coordinate system is afflicted with its own set of truncation errors, the implications of which must be understood and prioritized.

Isopycnic (potential density) coordinate modeling seeks to eliminate truncation errors by reversing the traditional role of depth as an independent variable and potential density as a dependent variable. More specifically, mixing in turbulent stratified fluids where buoyancy effects play a role takes place predominantly along isopycnic or constant potential density surfaces (Iselin 1939; Montgomery 1940; McDougall and Church 1986). If the conservation equations for salt and temperature are discretized in (x,y,z) space, that is, if the three-dimensional vectorial transport of these quantities is numerically evaluated as the sum of scalar transports in (x,y,z) direction, experience shows that it is virtually impossible to avoid diffusion of the transported variables in those three directions (Veronis 1975; Redi 1982; Cox 1987; Gent and McWilliams 1990, 1995; Griffies et al. 2000b). Thus, regardless of how the actual mixing term in the conservation equations is formulated, numerically induced mixing is likely to have a cross-isopycnal ("diapycnal") component that may well overshadow the common diapycnal processes that occur in nature (Griffies et al 2000b). This type of truncation error can be mostly eliminated in isopycnic coordinate modeling by transforming the dynamic equations from (x,y,z)

to (*x,y,ρ*) coordinates (Bleck 1978, 1998; Bleck et al. 1992; Bleck and Chassignet 1994).

First, the prognostic primitive equations are rewritten in (*x,y,s*) coordinates where *s* is an unspecified generalized vertical coordinate (Bleck 2002).

$$\frac{\partial \mathbf{v}}{\partial t_s} + \nabla_s \frac{\mathbf{v}^2}{2} + (\varsigma + f)\,\mathbf{k} \times \mathbf{v} + \left(\dot{s}\frac{\partial p}{\partial s}\right)\frac{\partial \mathbf{v}}{\partial p} = p\nabla_s \alpha - \nabla_\alpha M - g\frac{\partial \boldsymbol{\tau}}{\partial p} + \left(\frac{\partial p}{\partial s}\right)^{-1} \nabla_s \cdot \left(\nu \frac{\partial p}{\partial s} \nabla_s \mathbf{v}\right) \quad (11.1)$$

$$\frac{\partial}{\partial \mathbf{t}_s}\left(\frac{\partial p}{\partial s}\right) + \nabla_s \cdot \left(\mathbf{v}\frac{\partial p}{\partial s}\right) + \frac{\partial}{\partial s}\left(\dot{s}\frac{\partial p}{\partial s}\right) = 0 \quad (11.2)$$

$$\frac{\partial}{\partial \mathbf{t}_s}\left(\frac{\partial p}{\partial s}\theta\right) + \nabla_s \cdot \left(\mathbf{v}\frac{\partial p}{\partial s}\theta\right) + \frac{\partial}{\partial s}\left(\dot{s}\frac{\partial p}{\partial s}\theta\right) = \nabla_s \cdot \left(\mu \frac{\partial p}{\partial s}\nabla_s \theta\right) + H_\theta \quad (11.3)$$

where $\mathbf{v}$=(u,v) is the horizontal vector, p is pressure, θ represents any one of the model's thermodynamic variables, $\alpha = \rho_{\mathrm{pot}}^{-1}$ is the potential specific volume, $\varsigma = \partial v/\partial x_s - \partial u/\partial y_s$ is the relative vorticity, $M = gz + p\alpha$ is the Montgomery potential, *f* is the Coriolis parameter, **k** is the vertical unit vector, ν and μ are the eddy viscosity and diffusivity, τ is the wind- and/or bottom-drag induced shear stress vector, and H_θ is the sum of diabatic source terms acting on θ including diapycnal mixing. Subscripts indicate which variable is held constant during partial differentiation. Distances in the *x,y* direction, as well as their time derivatives *u,v*, are measured in the projection onto an horizontal plane. This conversion renders the coordinate system nonorthogonal in 3-D space, but eliminates metric terms related to the slope of the *s* surface (Bleck 1978). Other metric terms, created when vector products involving $(\nabla\cdot)$ or $(\nabla\times)$ are evaluated on a non Cartesian grid (e.g. spherical coordinates), are absorbed into the primary terms by evaluating vorticity and horizontal flux divergences in (11.1)–(11.3) as line integrals around individual grid boxes (see Griffies et al. 2000a for more details). Note that applying ∇ to a scalar, such as $\mathbf{v}^2/2$ in (11.1), does not give rise to metric terms.

Second, by performing a vertical integration over a coordinate layer bounded by two surfaces s_{top} and s_{bot}, the continuity Eq. (11.2) becomes a prognostic equation for the layer weight per unit area, $\Delta p = p_{\mathrm{bot}} - p_{\mathrm{top}}$.

$$\frac{\partial \Delta p}{\partial t_s} + \nabla_s \cdot (\mathbf{v}\Delta p) + \left(\dot{s}\frac{\partial p}{\partial s}\right)_{\mathrm{bot}} - \left(\dot{s}\frac{\partial p}{\partial s}\right)_{\mathrm{top}} = 0 \quad (11.4)$$

The expression $(\dot{s}\partial p/\partial s)$ represents the vertical mass flux across an *s* surface, taken to be positive in the +*p* downward direction. Multiplication of (11.1) by $\partial p/\partial s$ and integration over the interval (s_{top}, s_{bot}), followed by division by $\Delta p/\Delta s$, changes the shear stress term to $g/\Delta p(\tau_{\mathrm{top}} - \tau_{\mathrm{bot}})$ while the lateral momentum mixing term integrates to $(\Delta p)^{-1}\nabla_s\cdot(\nu\Delta p\nabla_s\mathbf{v})$. All the other terms in (11.1) retain their formal appearance. The layer integrated form of (11.3) is

$$\frac{\partial}{\partial \mathrm{t_s}}(\Delta p\theta) + \nabla_s\cdot(\mathbf{v}\Delta p\theta) + \left(\dot{s}\frac{\partial p}{\partial s}\theta\right)_{\mathrm{bot}} - \left(\dot{s}\frac{\partial p}{\partial s}\theta\right)_{\mathrm{top}} = \nabla_s\cdot(\mu\Delta p\nabla_s\theta) + \mathrm{H}_\theta \quad (11.5)$$

The above prognostic equations are complemented by several diagnostic equations, including the hydrostatic equation, $\partial M/\partial\alpha = p$, an equation of state linking potential temperature *T*, salinity *S*, and pressure *p* to ρ_{pot}, and an equation prescribing the vertical mass flux $(\dot{s}\partial p/\partial s)$ through an *s* surface.

Isopycnic models solve the above equations by using the potential density, ρ_{pot}, as the vertical coordinate *s*. Where the fluid is adiabatic, potential density is conserved and transport in the *x,y* direction then takes place in the model on isopycnic surfaces. This makes isopycnic models very adiabatic and allows them to avoid introducing the numerical diffusion in the vertical that can be troublesome in *z* or σ coordinate models. Transport in the *z* direction translates into transport along the ρ_{pot} axis and can be entirely suppressed if so desired; that is, it has no unwanted diapycnal component. As a result, spurious heat exchange between warm surface waters and cold abyssal waters and horizontal heat exchange across sloping isopycnals such as those marking frontal zones are minimized. Potential density surfaces are not, however, neutral surfaces (McDougall and Church 1986) and dianeutral fluxes may therefore be present when potential density is used as the vertical coordinate. As coordinate surfaces deviate from neutral, advection and diffusion acting along these surfaces will induce some dianeutral mixing. The impact of dianeutral fluxes from diffusion can be reduced/eliminated by rotating the diffusion operator to act along neutral directions in a manner analogous to that employed in fixed coordinates models (Griffies et al. 2000a). Furthermore, the nonlinear equation of state in the ocean introduces new physical sources of mixing, i.e., the independent transport of two active tracers (temperature and salinity) requires remapping algorithms to retain fields within pre-specified density classes. The level of dianeutral mixing introduced by remapping algorithms is usually negligible, but it has yet to be systematically documented (Griffies and Adcroft 2008). A reference pressure of 2000 db is now the norm for isopycnic coordinates models since it leads to few regions with coordinate inversions and the slopes of the σ_2 (potential density referred to 2000 db) surfaces are closest to neutral surfaces. Inclusion of thermobaricity (i.e., compressibility of sea water in the equation of state) in isopycnic coordinate ocean models is described in Sun et al. (1999) and Hallberg (2005). The necessity to properly reconcile estimates of the free surface height when using mode-splitting time stepping scheme is discussed in Hallberg and Adcroft (2009).

The key advantages of isopycnic coordinate models can be summarized as follows: (a) they are well suited for representing tracer transport without any large numerically induced vertical mixing as long as the isopycnals are reasonably parallel to neutral surfaces; (b) they conserve density classes under adiabatic motions; (c) the bottom topography is represented in a piecewise linear fashion, hence avoiding the need to distinguish bottom from side as traditionally done in z coordinate models; and (d) the overflows are well represented. The main drawback of an isopycnic coordinate model is its inability to properly represent the surface mixed layer or the bottom boundary layer since these layers are mostly unstratified. Examples of isopycnal models are NLOM (Wallcraft et al 2003), MICOM (Bleck et al. 1992; Bleck and Chassignet 1994; Bleck 1998), HIM (Hallberg 1995, 1997), OPYC (Oberhuber 1993) and POSUM.

As already stated, none of three main vertical coordinates currently in use (z, isopycnal, or σ) provides universal utility, and hybrid approaches have been developed in an attempt to combine the advantages of different types of vertical coordinates in optimally simulating the ocean. The term "hybrid vertical coordinates" can mean different things to different people: it can be a linear combination of two or more conventional coordinates (Song and Haidvogel 1994; Ezer and Mellor 2004; Barron et al. 2006) or it can be truly generalized, i.e., aiming to mimic different types of coordinates in different regions of a model domain (Bleck 2002; Burchard and Beckers 2004; Adcroft and Hallberg 2006; Song and Hou 2006). Adcroft and Hallberg (2006) classify generalized coordinates ocean models as either a Lagrangian Vertical Direction (LVD) or an Eulerian Vertical Direction (EVD) models. In LVD models, the continuity (thickness tendency) equation is solved forward in time throughout the domain, while an Arbitrary Lagrangian-Eulerian (ALE) technique is used to re-map the vertical coordinate and maintain different coordinate types within the domain. This differs from the EVD models with fixed depth and terrain-following vertical coordinates that use the continuity equation to diagnose vertical velocity.

The hybrid or generalized coordinate ocean models that have much in common with isopycnal models and are classified as LVD models are POSEIDON (Schopf and Loughe 1995) and HYCOM (Bleck 2002; Chassignet et al. 2003; Halliwell 2004). Other generalized vertical coordinate models currently under development are HYPOP and GOLD. HYPOP is the hybrid version of POP and differs from HYCOM and POSEIDON in the sense that the momentum equations continue to be solved on z coordinates while the tracer equations are solved using an ALE scheme for the vertical coordinate. Such an approach allows the model to utilize depth as the vertical coordinate in the mixed layer while using a more Lagrangian (e.g. isopycnal) coordinate in the deep ocean. GOLD, the "Generalized Ocean Layer Dynamics" model, is intended to be the vehicle for the consolidation of all of the climate ocean model development efforts at GFDL, including MOM and HIM.

11.5 Application: The HYbrid Coordinate Ocean Model (HYCOM)

The generalized vertical coordinates in HYCOM deviate from isopycnals (constant potential density surfaces) wherever the latter may fold, outcrop, or generally provide inadequate vertical resolution in portions of the model domain.

HYCOM is at its core a Lagrangian layer model, except for the remapping of the vertical coordinate by the hybrid coordinate generator after all equations are solved (Bleck 2002; Chassignet et al. 2003; Halliwell 2004) and for the fact that there is a nonzero horizontal density gradient within all layers. HYCOM is thus classified as an LVD model. The ability to adjust the vertical spacing of the coordinate surfaces in HYCOM simplifies the numerical implementation of several physical processes (e.g., mixed layer detrainment, convective adjustment, sea ice modeling) without harming the model of the basic and numerically efficient resolution of the vertical that is characteristic of isopycnic models throughout most of the ocean's volume (Bleck and Chassignet 1994; Chassignet et al. 1996).

HYCOM is the result of a collaboration initiated in the late nineties by ocean modelers at the Naval Research Laboratory, Stennis, MS, who approached colleagues at the University of Miami's Rosenstiel School of Marine and Atmospheric Science regarding an extension of the range of applicability of the U.S. Navy operational ocean prediction system to coastal regions (e.g., the U.S. Navy systems at the time were seriously limited in shallow water and in handling the transition from deep to shallow water). HYCOM (Bleck 2002) was therefore designed to extend the range of existing operational Ocean General Circulation Models (OGCMs). The freedom to adjust the vertical spacing of the generalized (or hybrid) coordinate layers in HYCOM simplifies the numerical implementation of several processes and allows for a smooth transition from the deep ocean to coastal regimes. HYCOM retains many of the characteristics of its predecessor, MICOM, while allowing coordinates to locally deviate from isopycnals wherever the latter may fold, outcrop, or generally provide inadequate vertical resolution. The collaboration led to the development of a consortium for hybrid-coordinate data assimilative ocean modeling supported by NOPP to make HYCOM a state-of-the-art community ocean model with data assimilation capability that could (1) be used in a wide range of ocean-related research; (2) become the next generation eddy-resolving global ocean prediction system; and (3) be coupled to a variety of other models, including littoral, atmospheric, ice and bio-chemical. The HYCOM consortium became one of the U.S. components of GODAE, a coordinated international system of observations, communications, modeling, and assimilation that delivers regular, comprehensive information on the state of the oceans (see Chassignet and Verron (2006) for a review). Navy and NOAA applications such as maritime safety, fisheries, the offshore industry, and management of shelf/coastal areas are among the expected beneficiaries of the HYCOM ocean prediction systems (http://www.hycom.org). More specifically, the precise knowledge and prediction of ocean mesoscale features helps the Navy, NOAA, the Coast Guard, the oil industry, and fisheries with endeavours such as ship and submarine routing, search and rescue, oil spill drift prediction, open ocean ecosystem monitoring, fisheries management, and short-range coupled atmosphere-ocean, coastal and near-shore environment forecasting. In addition to operational eddy-resolving global and basin-scale ocean prediction systems for the U.S. Navy and NOAA, respectively, this project offered an outstanding opportunity for

NOAA-Navy collaboration and cooperation ranging from research to the operational level (see Chassignet et al. 2009).

11.5.1 Hybrid Coordinate Generator

In HYCOM, the optimal vertical coordinate distribution of the three vertical coordinate types (pressure, isopycnal, sigma) is chosen at every time step and in every grid column individually. The default configuration of HYCOM is isopycnic in the open stratified ocean, but it makes a dynamically and geometrically smooth transition to terrain-following coordinates in shallow coastal regions and to fixed pressure-level (mass conserving) coordinates in the surface mixed layer and/or unstratified open seas. In doing so, the model takes advantage of the different coordinate types in optimally simulating coastal and open-ocean circulation features (Chassignet et al. 2003, 2006, 2007, 2009). A user-chosen option allows specification of the vertical coordinate separation that controls the transition among the three coordinate systems (Chassignet et al. 2007). The assignment of additional coordinate surfaces to the oceanic mixed layer also allows the straightforward implementation of multiple vertical mixing turbulence closure schemes (Halliwell 2004). The choice of the vertical mixing parameterization is also of importance in areas of strong entrainment, such as overflows (Papadakis et al. 2003; Xu et al. 2006, 2007; Legg et al. 2009).

The implementation of the generalized vertical coordinate in HYCOM follows the theoretical foundation set forth in Bleck and Boudra (1981) and Bleck and Benjamin (1993): i.e., each coordinate surface is assigned a reference isopycnal. The model continually checks whether grid points lie on their reference isopycnals, and, if not, attempts to move them vertically toward the reference position. However, the grid points are not allowed to migrate when this would lead to excessive crowding of coordinate surfaces. Thus, vertical grid points can be geometrically constrained to remain at a fixed pressure depth while being allowed to join and follow their reference isopycnals in adjacent areas (Bleck 2002). After the model equations are solved, the hybrid coordinate generator then relocates vertical interfaces to restore isopycnic conditions in the ocean interior to the greatest extent possible, while enforcing the minimum thickness requirements between vertical coordinates (see Chassignet et al. (2007) for details). If a layer is less dense than its isopycnic reference density, the generator attempts to move the bottom interface downward so that the flux of denser water across this interface increases density. If the layer is denser than its isopycnic reference density, the generator attempts to move the upper interface upward to decrease density. In both cases, the generator first calculates the vertical distance over which the interface must be relocated so that volume-weighted density of the original plus new water in the layer equals the reference density. The minimum permitted thickness of each layer at each model grid point is then calculated using the criteria provided by the user and the final minimum thickness is then calculated using a "cushion" function (Bleck 2002) that produces a smooth transition from the isopycnic to the p and σ domains. The minimum thickness constraint is not enforced at the bottom in the open ocean, permitting the model layers to collapse to zero thickness there, as in MICOM. Repeated execution of this algorithm at every time step maintains layer density very close to its reference value as long as a minimum thickness does not have to be maintained and diabatic processes are weak. To ensure that a permanent p coordinate domain exists near the surface year round at all model grid points, the reference densities of the uppermost layers are assigned values smaller than any density values found in the model domain.

Figure 11.2 illustrates the transition that occurs between p/σ and isopycnic (ρ_{pot}) coordinates in the fall and spring in the upper 400 m and over the shelf in the East

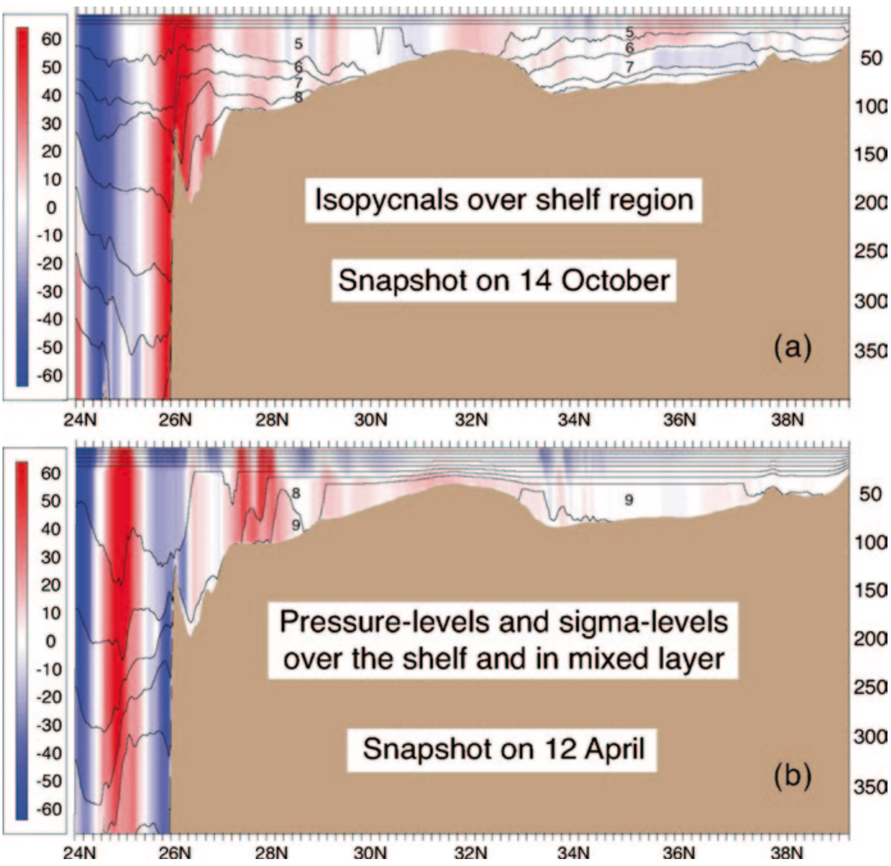

Fig. 11.2 Upper 400 m north-south velocity cross-section along 124.5°E in a 1/25° East China and Yellow Seas HYCOM embedded in a 1/6° North Pacific configuration forced with climatological monthly winds. **a** In the fall, the water column is stratified over the shelf and can be represented with isopycnals (ρ_{pot}). **b** In the spring, the water column is homogenized over the shelf and the vertical coordinate becomes a mixture of pressure (p) levels and terrain-following (σ) levels. The isopycnic layers are numbered over the shelf; the higher the number, the denser the layer. (From Chassignet et al. 2007)

China and Yellow Seas. In the fall, the water column is stratified and can be largely represented with isopycnals; in the spring, the water column is homogenized over the shelf and is represented by a mixture of p and σ coordinates. A particular advantage of isopycnic coordinates is illustrated by the density front formed by the Kuroshio above the peak of the sharp (lip) topography at the shelfbreak in Fig. 11.2a. Since the lip topography is only a few grid points wide, this topography and the associated front is best represented in isopycnic coordinates. In other applications in the coastal ocean, it may be more desirable to provide high resolution from surface to bottom to adequately resolve the vertical structure of water properties and of the bottom boundary layer. Since vertical coordinate choices for open-ocean HYCOM runs typically maximize the fraction of the water column that is isopycnic, it is often necessary to add more layers in the vertical-to-coastal HYCOM simulations nested within larger-scale HYCOM runs. An example using nested West Florida Shelf simulations (Halliwell et al. 2009) is illustrated in the cross sections in Fig. 11.3. The original vertical discretization is compared to two others with six layers added at the top: one with p coordinates and the other with σ coordinates over the shelf. This illustrates the flexibility with which vertical coordinates can be chosen by the user.

Maintaining hybrid vertical coordinates can be thought of as upwind finite volume advection. The original grid generator (Bleck 2002) used the simplest possible scheme of this type, the 1st order donor-cell upwind scheme. A major advantage of this scheme is that moving a layer interface does not affect the layer profile in the down-wind (detraining) layer, which greatly simplifies re-mapping to isopycnal layers. However, the scheme is diffusive when layers are re-mapped (there is no diffusion when layer interfaces remain at their original location). Isopycnal layers require minimal re-mapping in response to weak interior diapycnal diffusivity, but fixed coordinate layers often require significant re-mapping, especially in regions with significant upwelling or downwelling. Therefore, to minimize diffusion associated with the remapping, the grid generator was first replaced by a piecewise linear method (PLM) with a monotonized central-difference (MC) limiter (van Leer 1977) for layers that are in fixed coordinates while still using donor-cell upwind for layers that are non-fixed (and hence tending to isopycnal coordinates). PLM replaces the "constant within each layer" profile of donor-cell with a linear profile that equals the layer average at the center of the layer. The slope must be limited to maintain monotonicity. There are many possible limiters, but the MC limiter is one of the more widely used (Leveque 2002). The most recent version of the grid generator uses a weighted essentially non-oscillatory (WENO)-like piecewise parabolic method (PPM) scheme for increased accuracy. The generator also has been modified for situations when there is a too-light layer on top of a too-dense layer, i.e., when each layers attempts to gain mass at the expense of the other. Previously the generator chose each layer half of the time, but in practice the thicker of the two layers tended to gain mass and over time, the thinner layer tended to become very thin and stay that way. Now the thinner of the two layers always gains mass from the thicker layer, greatly reducing this tendency for layers to collapse.

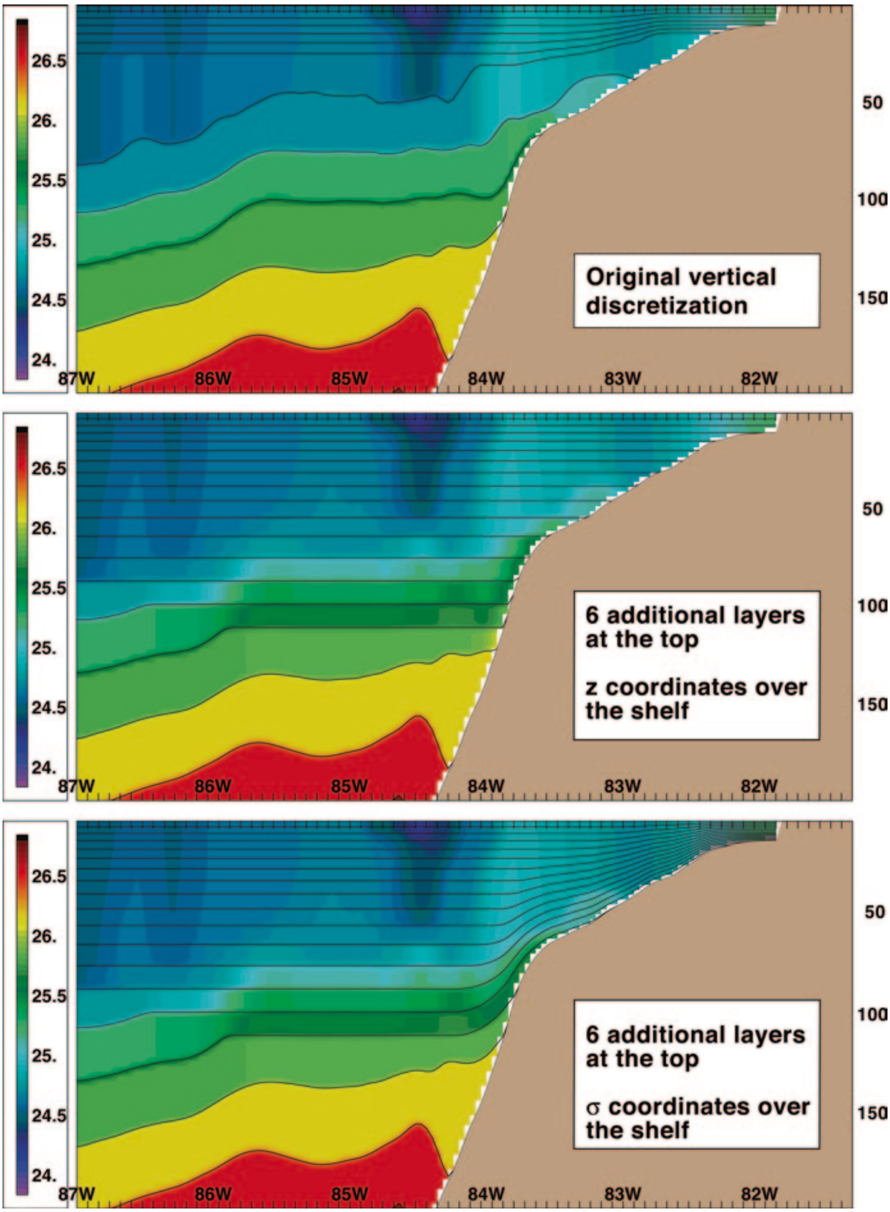

Fig. 11.3 Cross-sections of layer density and model interfaces across the West Florida Shelf in a 1/25° West Florida Shelf subdomain covering the Gulf of Mexico east of 87°W and north of 23°N (Halliwell et al. 2009). (From Chassignet et al. 2006, 2007)

11.5.2 The HYCOM Ocean Prediction Systems (http://www.hycom.org)

Data assimilation is essential for ocean prediction because (a) many ocean phenomena are due to nonlinear processes (i.e., flow instabilities) and thus are not a deterministic response to atmospheric forcing; (b) errors exist in the atmospheric forcing; and (c) ocean models are imperfect, including limitations in numerical algorithms and in resolution. Most of the information about the ocean surface's space-time variability is obtained remotely from instruments aboard satellites (i.e. sea surface height and sea surface temperature), but these observations are insufficient for specifying the subsurface variability. Vertical profiles from expendable bathythermographs (XBT), conductivity-temperature-depth (CTD) profilers, and profiling floats (e.g., Argo, which measures temperature and salinity in the upper 2000 m of the ocean) provide another substantial source of data. Even together, these data sets are insufficient to determine the state of the ocean completely, so it is necessary to use prior statistical knowledge based on past observations as well as our present understanding of ocean dynamics. By combining all of these observations through data assimilation into an ocean model, it is possible, in principle, to produce a dynamically consistent depiction of the ocean. However, to have any predictive capabilities, it is extremely important that the freely evolving ocean model (i.e., non-data-assimilative model) has skill in representing ocean features of interest.

To properly assimilate the SSH anomalies determined from satellite altimeter data, the oceanic mean SSH over the altimeter observation period must be provided. In this mean, it is essential that the mean current systems and associated SSH fronts be accurately represented in terms of position, amplitude, and sharpness. Unfortunately, the earth's geoid is not presently known with sufficient accuracy for this purpose, and coarse hydrographic climatologies (~0.5°–1° horizontal resolution) cannot provide the spatial resolution necessary when assimilating SSH in an eddy-resolving model (horizontal grid spacing of 1/10° or finer). At these scales of interest, it is essential to have the observed means of boundary currents and associated fronts sharply defined (Hurlburt et al. 2008). Figure 11.4 shows the climatological mean derived on a 0.5° grid using surface drifters by Maximenko and Niiler (2005) as well as a mean derived for the 1/12° Navy global HYCOM prediction system (see following section for details). The HYCOM mean was constructed as follows: a 5-year mean SSH field from a non-data assimilative 1/12° global HYCOM run was compared to available climatologies and a rubber-sheeting technique (Carnes et al. 1996) was used to modify the model mean in two regions (the Gulf Stream and the Kuroshio) where the western boundary current extensions were not well represented and where an accurate frontal location is crucial for ocean prediction. Rubber-sheeting involves a suite of computer programs that operate on SSH fields, overlaying contours from a reference field and moving masses of water in an elastic way (hence rubber-sheeting).

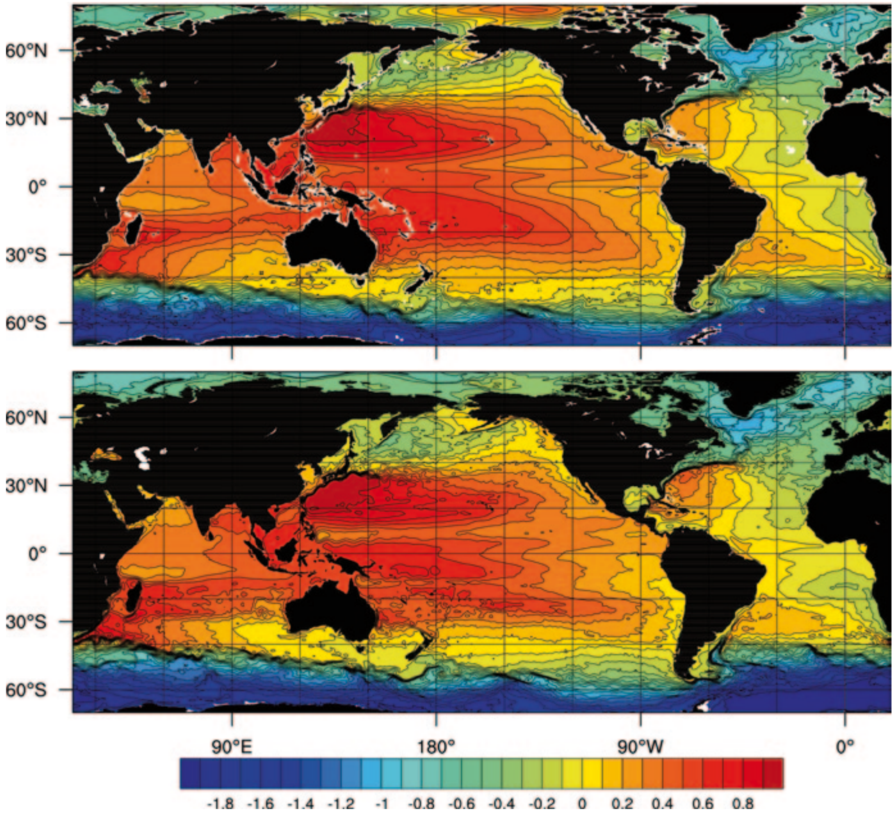

Fig. 11.4 Mean SSH (in cm) derived from surface drifters (Maximenko and Niiler 2005) (*top panel*) and from a non-data assimilative HYCOM run corrected in the Gulf Stream and Kuroshio regions using a rubber-sheeting technique (*bottom panel*). The RMS difference between the two fields is 9.2 cm. (From Chassignet et al. 2009)

Two systems are currently run in real-time by the U.S. Navy at NAVOCEANO, Stennis Space Center, MS, and by NOAA at NCEP, Washington, D.C. The first system is the NOAA Real Time Ocean Forecast System for the Atlantic (RTOFS-Atlantic), which has been running in real-time since 2005. The Atlantic domain spans 25°S–76°N with a horizontal resolution varying from 4 km near the U.S. coastline to 20 km near the African coast. The system is run daily with one-day nowcasts and five-day forecasts. Prior to June 2007, only the sea surface temperature was assimilated. In June 2007, NOAA implemented the 3D-Var data assimilation of (1) sea surface temperature and sea surface height (JASON, GFO, and soon ENVISAT), (2) temperature and salinity profile assimilation (ARGO, CTD, moorings, etc.), and (3) GOES data. Plans are to expand this system globally using the U.S Navy configuration described in the following paragraph. The NCEP

RTOFS-Atlantic model data is distributed in real time through NCEP's operational ftp server (ftp://ftpprd.ncep.noaa.gov) and the NOAA Operational Model Archive and Distribution System (NOMADS, http://nomads6.ncdc.noaa.gov/ncep_data/index.html) server. The latter server is also using OPeNDAP middleware as a data access method. NCEP's RTOFS-Atlantic model data is also archived at the National Oceanographic Data Center (NODC, http://data.nodc.noaa.gov/ncep/rtofs).

The second system is the global U.S. Navy nowcast/forecast system using the 1/12° global HYCOM (6.5 km grid spacing on average, 3.5 km grid spacing at North Pole, and 32 hybrid layers in the vertical), which has been running in near real-time since December 2006 and in real-time since February 2007. The current ice model is thermodynamic, but it will soon include more physics as it is upgraded to the Polar Ice Prediction System (PIPS, based on the Los Alamos CICE ice model). The model is currently running daily at NAVOCEANO using a part of the operational allocation on the machine. The daily run consists of a 5-day hindcast and a 5-day forecast. The system assimilates (1) SSH (Envisat, GFO, and Jason-1), (2) SST (all available satellite and in-situ sources), (3) all available in-situ temperature and salinity profiles (ARGO, CTD, moorings, etc.), and (4) SSMI sea ice concentration. The three-dimensional multivariate optimum interpolation Navy Coupled Ocean Data Assimilation (NCODA) (Cummings 2005) system is the assimilation technique. The NCODA horizontal correlations are multivariate in geopotential and velocity, thereby permitting adjustments (increments) to the mass field to be correlated with adjustments to the flow field. The velocity adjustments are in geostrophic balance with the geopotential increments, and the geopotential increments are in hydrostatic agreement with the temperature and salinity increments. Either the Cooper and Haines (1996) technique or synthetic temperature and salinity profiles (Fox et al. 2002) can be used for downward projection of SSH and SST. An example of forecast performance is shown in Fig. 11.5

Validation of the results is underway using independent data with a focus on the large-scale circulation features, SSH variability, eddy kinetic energy, mixed layer depth, vertical profiles of temperature and salinity, SST and coastal sea levels (Metzger et al. 2008). Figures 11.6 and 11.7 show examples for the Gulf Stream region while Fig. 11.8 documents the performance of HYCOM in representing the mixed layer depth. HYCOM is also an active participant in the international GODAE comparison of global ocean forecasting systems.

11.5.3 Distribution of Global HYCOM Hindcasts and Forecasts

The model outputs from the global U.S. Navy hindcast experiment from November 2003 to present are available through the HYCOM consortium web page, http://www.hycom.org. The HYCOM data distribution team developed and implemented a comprehensive data management and distribution strategy that allowed easy and

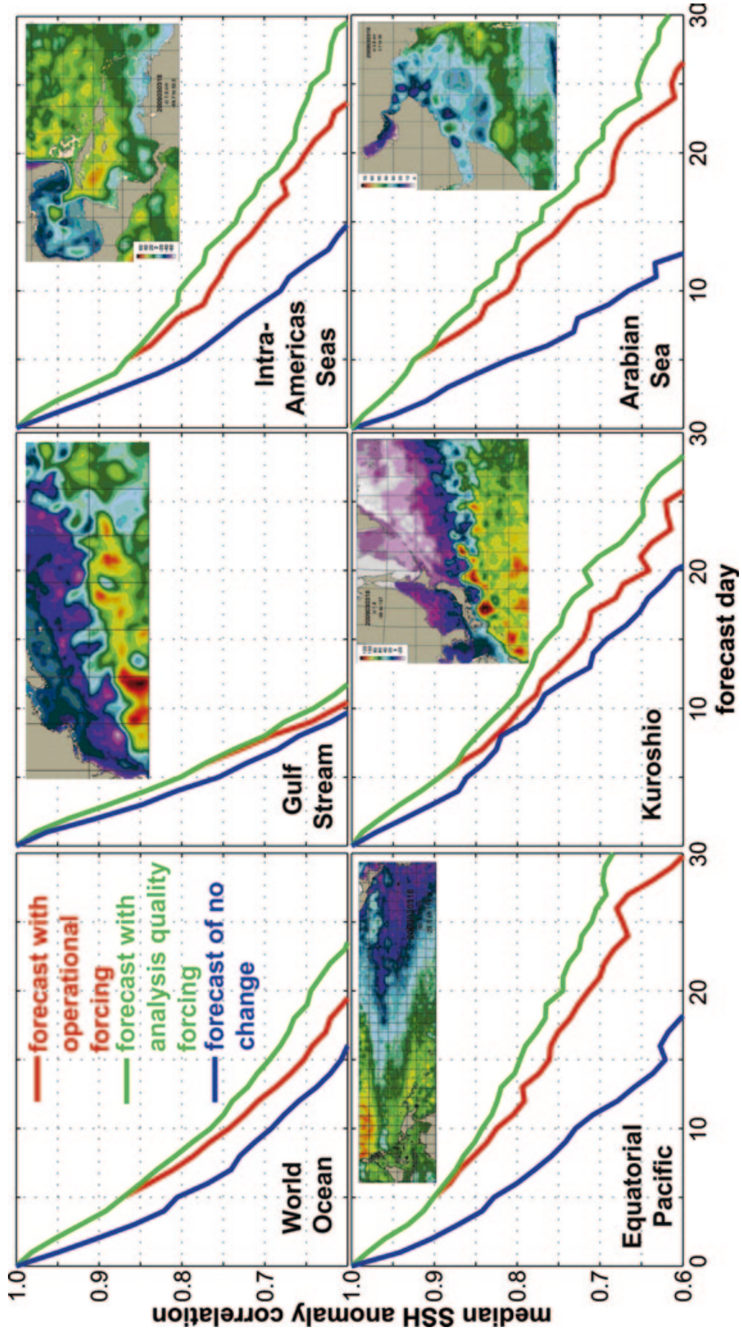

Fig. 11.5 Verification of 30-day ocean forecasts: median SSH anomaly correlation vs. forecast length in comparison with the verifying analysis for the global U.S Navy HYCOM over the world ocean and five subregions. The *red curves* verify forecasts using operational atmospheric forcing, which reverts toward climatology after five days. The *green curves* verify "forecasts" with analysis quality forcing for the duration and the *blue curves* verify forecasts of persistence (i.e., no change from the initial state). The plots show median statistics over twenty 30-day HYCOM forecasts initialized during January 2004—December 2005, a period when data from three nadir-beam altimeters, Envisat, GFO and Jason-1, were assimilated. The reader is referred to Hurlburt et al. (2008, 2009) for a more detailed discussion of these results. (From Chassignet et al. 2009)

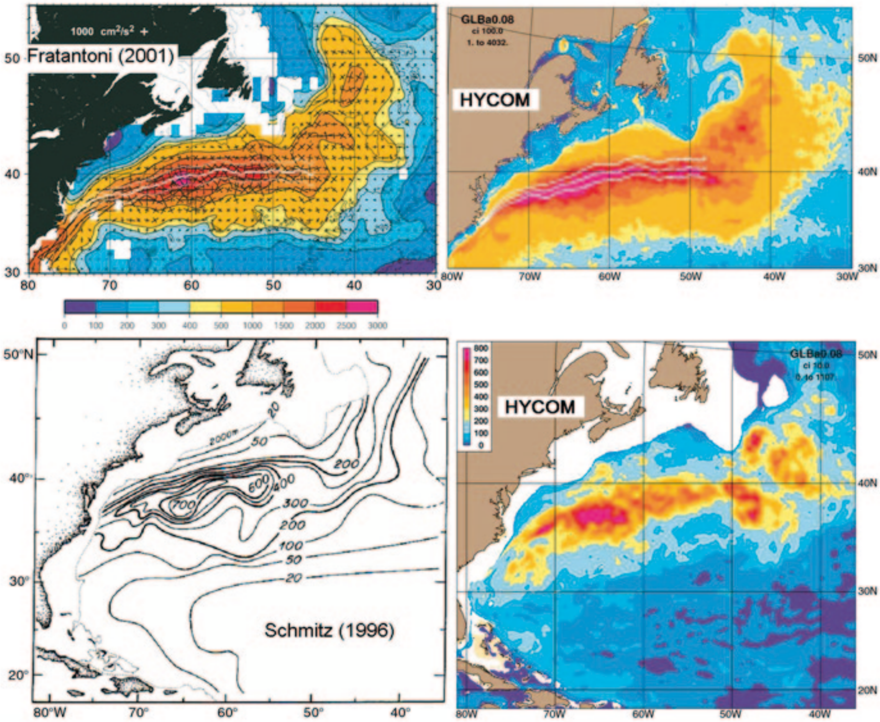

Fig. 11.6 Surface (*top panels*) and 700 m (*lower panels*) eddy kinetic energy from observations (*left panels*) and HYCOM over the period 2004–2006 (*right panels*). The observed surface eddy kinetic energy (*upper left panel*) is from Fratantoni (2001) and the 700 m eddy kinetic energy (*lower left panel)* is from Schmitz (1996). The units are in cm^2/s^2. Overlaid on the top panels is the Gulf Stream north wall position ±1 standard deviation. (From Chassignet et al. 2009)

efficient access to the global HYCOM-based ocean prediction system output to (a) coastal and regional modeling groups; (b) the wider oceanographic and scientific community, including climate and ecosystem researchers; and (c) the general public. The outreach system consists of a web server that acts as a gateway to backend data management, distribution, and visualization applications (http://www.hycom.org/dataserver). These applications enable end users to obtain a broad range of services such as browsing of datasets, GIF images, NetCDF files, FTP requests of data, etc. The 130 Terabytes HYCOM Data Sharing System is built upon two existing software components: the Open Project for a Network Data Access Protocol (OPeNDAP) (Cornillon et al. 2009) and the Live Access Server (LAS) (http://ferret.pmel.noaa.gov/LAS/). These tools and their data distribution methods are described below. In the current setup, the OPeNDAP component provides the middleware necessary to access distributed data, while the LAS functions as a user interface and a product server. The abstraction offered by the OPeNDAP server also makes it possible to define a virtual data set that LAS will act upon, rather than physical files. An

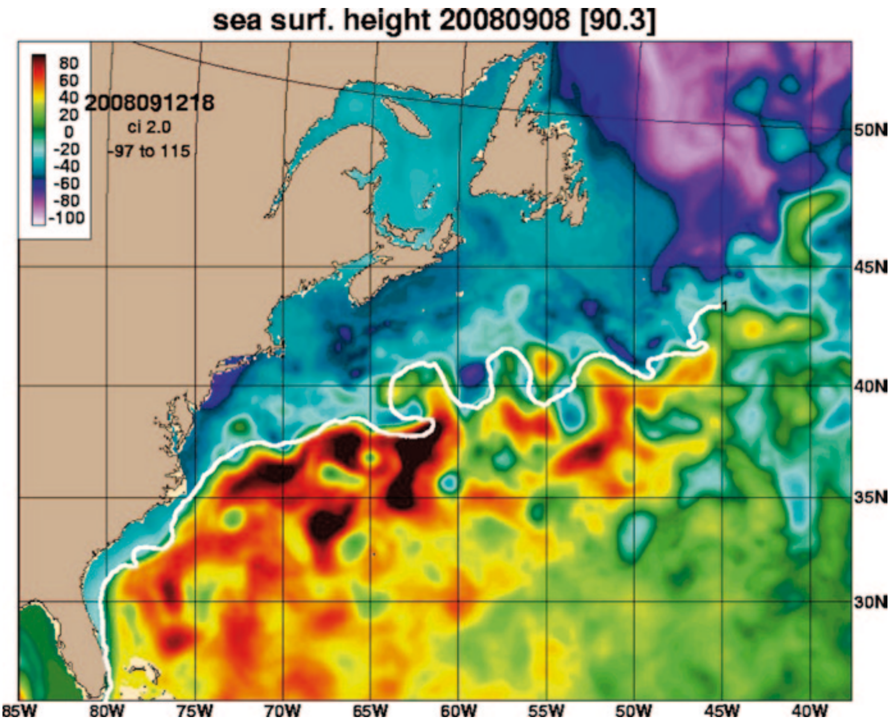

Fig. 11.7 Modeled analysis of the sea surface height field on September 8, 2008. The *white line* represents the independent frontal analysis of sea surface temperature observations performed by the Naval Oceanographic Office. (From Chassignet et al. 2009)

OPeNDAP "aggregation server" utilizes this approach to append model time steps from many separate files into virtual datasets. The HYCOM Data Service has been in operation for the last four years and has seen a steady increase in the user base. In the last year, the service received approximately 20,000 hits per month. In addition to the numerous requests from educational institutions and researchers, this service has been providing near real-time data products to several private companies in France, the Netherlands, Portugal, and the U.S.

11.5.4 Boundary Conditions for Regional and Coastal Models Nested in HYCOM

An important attribute of the data assimilative HYCOM system is its capability to provide boundary conditions to even higher resolution regional and coastal models. The current horizontal and vertical resolution of the global forecasting

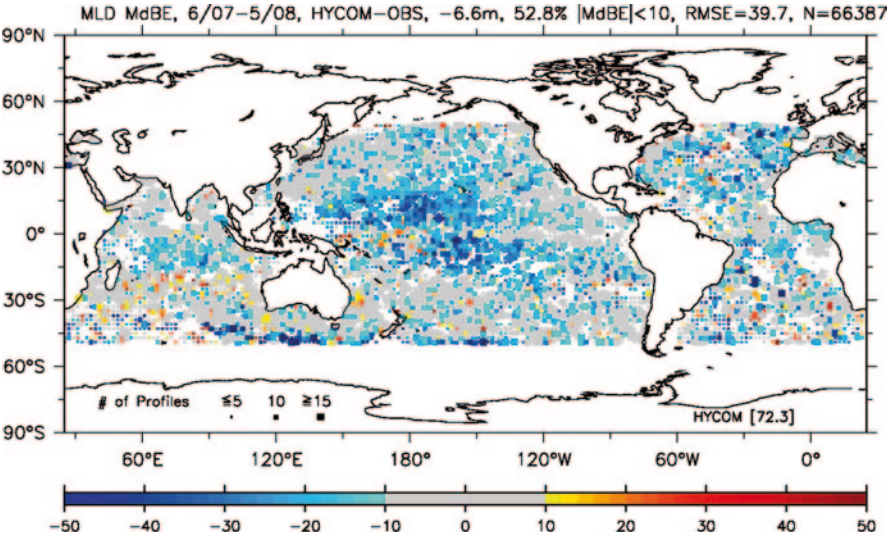

Fig. 11.8 Median bias error (in meters) of mixed layer depth (MLD) calculated from simulated and approximately 66000 unassimilated observed profiles over the period June 2007—May 2008. Blue (*red*) indicates a simulated MLD shallower (*deeper*) than observed; 53% of the simulated MLDs are within 10 m of the observation and these are represented as gray. The basin-wide median bias error is −6.6 m and the RMS error is 40 m. (From Chassignet et al. 2009)

system marginally resolves the coastal ocean (7 km at mid-latitudes, with up to 15 terrain-following (σ) coordinates over the shelf), but it is an excellent starting point for even higher resolution coastal ocean prediction efforts. Several partners within the HYCOM consortium evaluated the boundary conditions and demonstrated the value added by the global and basin HYCOM data assimilative system output for coastal ocean prediction models. The inner nested models may or may not be HYCOM (i.e., the nesting procedure can handle any vertical grid choice). Outer model fields are interpolated to the horizontal and vertical grid of the nested model throughout the entire time interval of the nested model simulation at a time interval specified by the user, typically once per day. The nested model is initialized from the first archive file and the entire set of archives provides boundary conditions during the nested run, ensuring consistency between initial and boundary conditions. This procedure has proven to be very robust. Figure 11.9 shows an example of the SST and surface velocity fields from a ROMS (Shchepetkin and McWilliams 2005) West Florida Shelf domain embedded in the U.S. Navy HYCOM ocean prediction system. The Gulf of Mexico Loop Current is the main large-scale ocean feature impacting the WFS and the impact of open boundary conditions on the dynamics and accuracy of the regional model was assessed by Barth et al. (2008). Examples can be found in Chassignet et al. (2006, 2009).

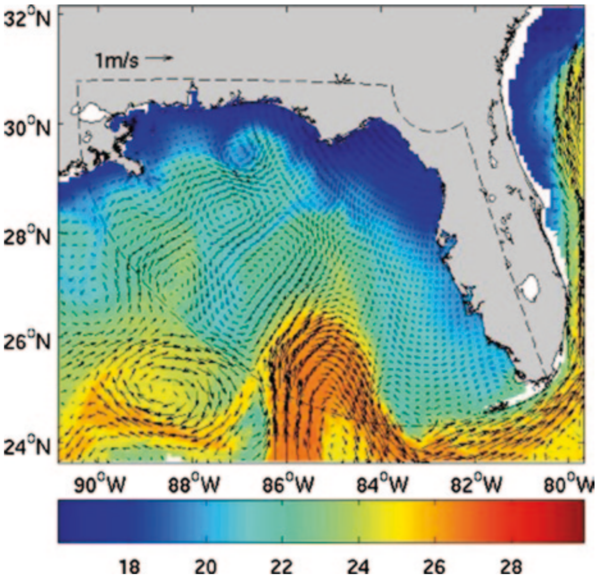

Fig. 11.9 Sea surface temperature (°C) and surface velocity fields from the ROMS West Florida Shelf domain (*inside the dashed lines*) and the HYCOM ocean prediction system (*outside the dashed lines*). (From Chassignet et al. 2009)

11.5.5 HYCOM Long-Term Development

The long-term goals of the HYCOM consortium for the global domain are to (a) add 3-D and 4-D VAR data assimilation, (b) increase the horizontal resolution of the global domain to 1/25°, (c) implement two-way nesting, (d) implement zero depth coastlines with wetting and drying, and (e) include tides. The scientific goals include, but are not be limited to (a) evaluation of the internal tides representation in support of field programs, (b) evaluation of the global model's ability to provide boundary conditions to very high resolution coastal models, (c) interaction of the open ocean with ice, (d) shelf-deep ocean interactions, (e) upper ocean physics including mixed layer/sonic depth representation, and (f) mixing processes. Other research activities will focus on coupled ocean-wave-atmosphere prediction, bio-geo-chemical-optical and tracer/contaminant prediction, ecosystem analysis and prediction, and earth system prediction (i.e., coupled atmosphere-ocean-ice-land).

11.6 Outlook

One of the greatest uncertainties in setting up a data assimilative system is the error one needs to attribute to the numerical model. To a certain extent, the rate at which a model moves away from the assimilative state will provide some indication of the model's performance. A careful comparison with observations in assessing the model's performance with and without data assimilation will help in identifying the model biases and the areas that need major improvements, either in representation or in parameterization. The routine analysis of model forecasts will provide a wealth of information the modeler can use to improve the model's physics, especially if additional forecasts/hindcasts can be performed after the fact to assess the effectiveness of the changes.

Much of the uncertainty associated with ocean prediction can be ascribed to an imperfect knowledge of the ocean and its mechanisms for mitigating or exacerbating changes in the atmosphere and cryosphere. Oceanic predictions rely both on the ability to initialize a model to agree with observed conditions and on the model's ability to accurately evolve this initial state. There are several classes of numerical circulation models that have achieved a significant level of community management and involvement, including shared development, regular user interaction, and ready availability of software and documentation via the worldwide web. The numerical codes are typically maintained within university user groups or by government laboratories; their development primarily resulted from individual efforts, rather than a cohesive community effort. While this was appropriate in a previous era when oceanic modeling was a smaller enterprise and computer architectures were simpler, the limitations of this approach are increasingly apparent in an era when many diverse demands are being placed upon oceanic models. The time has come for the development of modeling capabilities to become a coherent community effort that both systematically advances the models and supports widespread access. It is also important to state that one cannot separate the effective development of oceanic ecological and biogeochemical models from that of the physical circulation model, and these extended modeling capabilities need to be an integral part of this community effort. Moreover, testing of these models with observations requires advanced inverse methods and data assimilation techniques that must be linked to this effort from the outset.

Deliberations among physical and biogeochemical modelers led to the submission in 2006 to the U.S. National Science Foundation of a white paper entitled "Enabling a Community Environment for Advanced Oceanic Modeling", written by E. Chassignet, S. Doney, R. Hallberg, D. McGillicuddy, and J. McWilliams. It described issues confronted by a large and growing fraction of the ocean modeling community concerning the complexity and redundancy in ocean model development. It also outlined a long-term vision and course of action to address these concerns by proposing the development of a Community Environment for Advanced Ocean Modeling. Such an environment would:

- Create a common code base to allow a synthesis of different algorithmic elements.
- Provide a test bed to allow for exploration of the merits of different approaches in representing the important model elements, leading to recommendations for best practices.
- Provide estimates of model uncertainty by performing, for a given configuration, ensemble calculations with a variety of algorithms, vertical-coordinate and other discretizations, parameterizations, etc.

- Include the core algorithms for evaluation of current practices in marine ecological and biogeochemical modeling.
- Make available standard data sets to facilitate comparison to observations as well as algorithm development and testing.
- Facilitate linkage with inverse methods for testing models with observations, as well as data assimilation techniques for use in prediction and in the state estimation problem.
- Encourage collaboration among model developers to accelerate the pace of designing and testing new algorithms.
- Provide rapid community access to model advancements.

It is important to make the distinction between the proposed community ocean modeling environment and a single community ocean model. The proposed modeling environment would provide a common, interchangeable code base with minimized restrictions on the algorithms that can be contributed or selected for a specific model application. Whereas many model algorithm developers would find a single community ocean model to be stifling, a community modeling environment should dramatically invigorate the development of new and superior ocean modeling techniques. This environment will offer a much broader range of options than would be possible with a single monolithic model. This diversity of options is critical for selecting the most appropriate configuration for any particular oceanic application.

The 10-year vision is to have a broad unification of physical, ecological, and biogeochemical oceanic modeling tools and practices by collecting the expertise of the current sigma, geopotential, and isopycnic/hybrid vertical-coordinate models in a single open and multi-disciplinary software framework. This will allow the greatest possible flexibility for users and synergies for model developers. The environment will promote exploration of novel modeling concepts; more rapid improvement of multi-scale physical, ecological and biogeochemical models; and a stable base for the development of new application services built around a core model framework that can be maintained at the cutting edge of the science. It will also provide a framework for experimentation and rapid implementation of improvements in the parameterization of unresolved processes in oceanic models. The environment will furnish the capability to interchange, combine, and modify choices of vertical coordinate, physical parameterizations, numerical algorithms, parameter settings, and so on. This is in contrast with the usual single model consisting most of the time of a fixed set of parameterizations and algorithms, perhaps with some restricted freedom in the setting of parameters, but with very limited user options to experiment with the model architecture. It is indeed essential to maintain and extend the diversity of available algorithms. The diverse collection of techniques is the gene pool of future oceanic models, and a rich pool provides the best prospect for selecting the models that are optimal for answering specific questions about processes of interest. By comparing the performance of a rich array of configurations, the community will then be able to breed oceanic models that are most skillful at representing the broad assortment of processes important in the simulation of a system as complicated as the ocean. It will also provide an estimate of the model uncertainty by giving an envelope of solutions resulting from different choices in numerical algorithms; vertical, horizontal, and temporal discretizations; and parameterizations.

Acknowledgements As stated in the introduction, a lot of material presented in this chapter relies heavily on articles, notes, and review papers by R. Bleck, S. Griffies, A. Adcroft, and R. Hallberg. I also would like to acknowledge contributions by H. Hurlburt and B. Arbic. The development of the HYCOM ocean prediction system was sponsored by the National Oceanographic Partnership Program (NOPP) and the Office of Naval Research (ONR).

References

Adcroft A, Hallberg R (2006) On methods for solving the oceanic equations of motion in generalized vertical coordinates. Ocean Model 11:224–233
Arbic BK, Wallcraft AJ, Metzger EJ (2010) Concurrent simulation of the eddying general circulation and tides in a global ocean model. Ocean Model 32:175–187
Barron CN, Martin PJ, Kara AB, Rhodes RC, Smedstad LF (2006) Formulation, implementation and examination of vertical coordinate choices in the Global Navy Coastal Ocean Model (NCOM). Ocean Model 11:347–375
Barth A, Alvera-Azcárate A, Weisberg RH (2008) Benefit of nesting a regional model into a large-scale ocean model instead of climatology. Application to the West Florida Shelf. Cont Shelf Res 28:561–573
Bleck R (1978) Finite difference equations in generalized vertical coordinates. Part I: Total energy conservation. Contrib Atmos Phys 51:360–372
Bleck R (1998) Ocean modeling in isopycnic coordinates. In: Chassignet EP, Verron J (eds) Ocean Modeling and Paramterization. NATO Science Series. Kluwer Academic Publishers, Dordrecht, pp 423–448
Bleck R (2002) An oceanic general circulation model framed in hybrid isopycnic-cartesian coordinates. Ocean Model 4:55–88
Bleck R, Benjamin S (1993) Regional weather prediction with a model combining terrain-following and isentropic coordinates. Part I: Model description. Mon Wea Rev 121:1770–1785
Bleck R, Boudra D (1981) Initial testing of a numerical ocean circulation model using a hybrid (quasi-isopycnic) vertical coordinate. J Phys Oceanogr 11:755–770
Bleck R, Chassignet EP (1994) Simulating the oceanic circulation with isopycnic coordinate models. In: Majundar SK, Mill EW, Forbes GS, Schmalz RE, Panah AA (eds) The oceans: Physical-chemical dynamics and human impact. The Pennsylvania Academy of Science, Easton, pp 17–39
Bleck R, Rooth C, Hu D, Smith LT (1992) Salinity-driven thermocline transients in a wind- and thermohaline-forced isopycnic coordinate model of the North Atlantic. J Phys Oceanogr 22:1486–1505
Burchard H, Beckers JM (2004) Non-uniform adaptive vertical grids in one-dimensional numerical ocean models. Ocean Model 6:51–81
Carnes MR, Fox DN, Rhodes RC, Smedstad OM (1996) Data assimilation in a North Pacific Ocean monitoring and prediction system. In: Malanotte-Rizzoli P (ed) Modern approaches to data assimilation in ocean modeling. Elsevier, New York, pp 319–345
Chassignet EP, Garraffo ZD (2001) Viscosity parameterization and the Gulf Stream separation. In: Muller P, Henderson D (eds) From stirring to mixing in a stratified ocean. Proceedings 'Aha Huliko'a Hawaiian Winter Workshop. University of Hawaii. 15–19 January 2001, pp 37–41
Chassignet EP, Malanotte-Rizzoli P (2000) Ocean circulation model evaluation experiments for the North Atlantic basin. Dyn Atmos Oceans 32:155–432 (Elsevier Science Ltd., special issue)

Chassignet EP, Marshall DP (2008) Gulf Stream separation in numerical ocean models. In: Hecht M, Hasumi H (eds) Eddy-Resolving Ocean Modeling. AGU monograph series, American Geophysical Union, Washington, DC, pp 39–62
Chassignet EP, Verron J (1998) Ocean modeling and parameterization. Kluwer Academic Publishers, Dordrecht, p 451
Chassignet EP, Verron J (2006) Ocean weather forecasting: an integrated view of oceanography. Springer, Dordrecht, p 577
Chassignet EP, Smith LT, Bleck R, Bryan FO (1996) A model comparison: Numerical simulations of the North and Equatorial Atlantic Ocean circulation in depth and isopycnic coordinates. J Phys Oceanogr 26:1849–1867
Chassignet EP, Smith LT, Halliwell GR, Bleck R (2003) North Atlantic simulations with the HYbrid Coordinate Ocean Model (HYCOM): Impact of the vertical coordinate choice, reference density, and thermobaricity. J Phys Oceanogr 33:2504–2526
Chassignet EP, Hurlburt HE, Smedstad OM, Halliwell GR, Wallcraft AJ, Metzger EJ, Blanton BO, Lozano C, Rao DB, Hogan PJ, Srinivasan A (2006) Generalized vertical coordinates for eddy-resolving global and coastal ocean forecasts. Oceanography 19:20–31
Chassignet EP, Hurlburt HE, Smedstad OM, Halliwell GR, Hogan PJ, Wallcraft AJ, Baraille R, Bleck R (2007) The HYCOM (HYbrid Coordinate Ocean Model) data assimilative system. J Mar Sys 65:60–83
Chassignet EP, Hurlburt HE, Metzger EJ, Smedstad OM, Cummings J, Halliwell GR, Bleck R, Baraille R, Wallcraft AJ, Lozano C, Tolman HL, Srinivasan A, Hankin S, Cornillon P, Weisberg R, Barth A, He R, Werner F, Wilkin J (2009) U.S. GODAE: Global ocean prediction with the HYbrid Coordinate Ocean Model (HYCOM). Oceanography 22(2):64–75
Clarke AJ (2008) An introduction to the dynamics of El Niño & the Southern oscillation. Amsterdam, Elsevier, p 324
Cooper M, Haines K (1996) Altimetric assimilation with water property conservation. J Geophys Res 101:1059–1078
Cornillon P, Adams J, Blumenthal MB, Chassignet EP, Davis E, Hankin S, Kinter J, Mendelssohm R, Potemra JT, Srinivasan A, Sirott J (2009) NVODS and the development of OPeNDAP—an integrative tool for oceanographic data systems. Oceanography 22(2):116–127
Cox MD (1987) Isopycnal diffusion in a *z*-coordinate ocean model. Ocean Modelling (unpublished manuscripts) 74:1–5
Cummings JA (2005) Operational multivariate ocean data assimilation. Quart J Royal Met Soc 131:3583–3604
Ezer T, Mellor G (2004) A generalized coordinate ocean model and a comparison of the bottom boundary layer dynamics in terrainfollowing and z-level grids. Ocean Model 6:379–403
Fox DN, Teague WJ, Barron CN, Carnes MR, Lee CM (2002) The modular ocean data analysis system (MODAS). J Atmos Ocean Technol 19:240–252
Fratantoni DM (2001) North Atlantic surface circulation during the 1990's observed with satellite-tracked drifters. J Geophys Res 106:22,067–22,093
Gent PR, McWilliams JC (1990) Isopycnic mixing in ocean circulation models. J Phys Oceanogr 20:150–155
Gent PR, Willebrand J, McDougall TJ, McWilliams JC (1995) Parameterizing eddy-induced tracer transports in ocean circulation models. J Phys Oceanogr 25:463–474
Goff JA, Arbic BK (2010) Global prediction of abyssal hill roughness statistics for use in ocean models from digital maps of paleo-spreading rate, paleoridge orientation, and sediment thickness. Ocean Model 32:36–43. doi:10.1016/j.ocemod.2009.10.001
Greatbatch RJ, Mellor GL (1999) An overview of coastal ocean models. In: Mooers CNK (eds) Coastal ocean prediction. American Geophysical Union, Washington, pp 31–57, 526 pages total
Griffes SM, Adcroft AJ (2008) Formulating the equations of ocean models. In: Hecht M, Hasumi H (eds) Eddy resolving ocean modeling. Geophysical Monograph Series. American Geophysical Union, Washington, pp 281–318
Griffies SM, Hallberg RW (2000) Biharmonic friction with a Smagorinsky viscosity for use in large-scale eddy-permitting ocean models. Mon Weather Rev 128:2935–2946

Griffies SM, Böning C, Bryan FO, Chassignet EP, Gerdes R, Hasumi H, Hirst A, Treguier A-M, Webb D (2000a) Developments in ocean climate modelling. Ocean Model 2:123–192
Griffies SM, Pacanowski RC, Hallberg RW (2000b) Spurious diapycnal mixing associated with advection in a *z*-coordinate ocean model. Monthly Weather Rev 128:538–564
Hallberg RW (1995) Some aspects of the circulation in ocean basins with isopycnals intersecting the sloping boundaries, Ph.D. thesis, University of Washington, Seattle, p 244
Hallberg RW (1997) Stable split time stepping schemes for large-scale ocean modelling. J Comput Phys 135:54–65
Hallberg RW (2005) A thermobaric instability of Lagrangian vertical coordinate ocean models. Ocean Model 8:279–300
Hallberg RW, Adcroft A (2009) Reconciling estimates of the free surface height in Lagrangian vertical coordinate ocean models with mode-split time stepping. Ocean Model 29:15–26
Halliwell G (2004) Evaluation of vertical coordinate and vertical mixing algorithms in the HYbrid Coordinate Ocean Model (HYCOM). Ocean Model 7:285–322
Halliwell GR Jr, Barth A, Weisberg RH, Hogan P, Smedstad OM, Cummings J (2009) Impact of GODAE products on nested HYCOM simulations of the West Florida Shelf. Ocean Dyn 59:139–155
Hecht MW, Hasumi H (2008) Ocean modeling in an eddying regime. Geophysical monograph series, vol 7. American Geophysical Union, Washington, p 409
Hecht MW, Hunke E, Maltrud ME, Petersen MR, Wingate BA (2008) Lateral mixing in the eddying regime and a new broad-ranging formulation. In: Hecht, Hasumi (eds) Ocean modeling in an eddying regime. AGU Monograph Series. AGU, Washington, pp. 339–352
Hurlburt HE, Chassignet EP, Cummings JA, Kara AB, Metzger EJ, Shriver JF, Smedstad OM, Wallcraft AJ, Barron CN (2008) Eddy-resolving global ocean prediction. In: Hecht M, Hasumi H (ed) “Ocean Modeling in an Eddying Regime”. Geophysical monograph 177. American Geophysical Union, Washington, pp 353–381
Hurlburt HE, Brassington GB, Drillet Y, Kamachi M, Benkiran M, Bourdalle-Badie R, Chassignet EP, LeGalloudec O, Lellouche JM, Metzger EJ, Oke PR, Pugh T, Schiller A, Smedstad OM, Tranchant B, Tsujino H, Usui N, Wallcraft AJ (2009) High resolution global and basin-scale ocean analyses and forecasts. Oceanography 22(3):110–127
Iselin CO (1939) The influence of vertical and lateral turbulence on the characteristics of the waters at mid-depths. Eos Trans Am Geophys Union 20:414–417
Jakobsson M, Cherkis N, Woodward J, Coakley B, Macnab R (2000) A new grid of Arctic bathymetry: A significant resource for scientists and mapmakers, EOS Transactions. Am Geophys Union 81(9):89, 93, 96
Ledwell JR, Watson AJ, Law CS (1993) Evidence for slow mixing across the pycnocline from an open-ocean tracer-release experiment. Nature 364:701–703
Lee MM, Coward AC, Nurser AJ (2002) Spurious diapycnal mixing of the deep waters in an eddy-permitting global ocean model. J Phys Oceanogr 32:1522–1535
Legg S, Chang Y, Chassignet EP, Danabasoglu G, Ezer T, Gordon AL, Griffes S, Hallberg R, Jackson L, Large W, Özgökmen T, Peters H, Price J, Riemenschneider U, Wu W, Xu X, Yang J (2009) Improving oceanic overflow representation in climate models: the Gravity Current Entrainment Climate Process Team. Bull Am Met Soc 90(4):657–670. doi:10.1175/2008BAMS2667.1
Leveque, R.J. (2002) Finite volume methods for hyperbolic problems. Cambridge University Press, Cambridge, p 578
Maltrud ME, McClean JL (2005) An eddy resolving global 1/10_ ocean simulation. Ocean Model 8:31–54
Maximenko NA, Niiler PP (2005). Hybrid decade-mean sea level with mesoscale resolution. In: Saxena N (ed) “Recent Advances in Marine Science and Technology”. PACON International, Honolulu, pp. 55–59
McDougall TJ, Church JA (1986) Pitfalls with the numerical representation of isopycnal and diapycnal mixing. J Phys Oceanogr 16:196–199

Meincke JC, Le Provost, Willebrand J (2001) Dynamics of the North Atlantic Circulation (DYNAMO). Prog Oceanogr 48:N°2–3

Metzger EJ, Smedstad OM, Thoppil P, Hurlburt HE, Wallcraft AJ, Franklin DS, Shriver JF, Smedstad LF (2008) Validation Test Report for Global Ocean Prediction System V3.0—1/12°HYCOM/NCODA: Phase I, NRL Memo. Report, NRL/MR/7320—08-9148

Montgomery RB (1940) The present evidence on the importance of lateral mixing processes in the ocean. Bull Am Meteor Soc 21:87–94

Oberhuber JM (1993) Simulation of the atlantic circulation with a coupled sea ice-mixed layer-isopycnal general circulation model. Part I: model description. J Phys Oceanogr 23:808–829

Papadakis MP, Chassignet EP, Hallberg RW (2003) Numerical simulations of the Mediterranean Sea outflow: Impact of the entrainment parameterization in an isopycnic coordinate ocean model. Ocean Model 5:325–356

Philander SGH (1990) El Niño, La Niña, and the Southern Oscillation. Academic Press, New York, p 293

Redi MH (1982) Oceanic mixing by coordinate rotation. J Phys Oceanogr 12:87–94

Schmitz WJ (1996) On the World Ocean Circulation. Vol. 1: Some global features/North Atlantic circulation. Woods Hole Oceanographic Institute Tech. Rep. WHOI-96–03. p 141

Schopf PS, Loughe A (1995) A reduced-gravity isopycnal ocean model: Hindcasts of El Niño. Mon Wea Rev 123:2839–2863

Shchepetkin AF, McWilliams JC (2005) The Regional Ocean Modeling System (ROMS): A split-explicit, free-surface, topography-following coordinates ocean model. Ocean Model 9:347–404

Shriver JF, Hurlburt HE (2000) The effect of upper ocean eddies on the non-steric contribution to the barotropic mode. Geophys Res Lett 27:2713–2716

Shriver JF, Hurlburt HE, Smedstad OM, Wallcraft AJ, Rhodes RC (2007) 1/32°real-time global ocean prediction and value-added over 1/16 resolution. J Mar Sys 65:3–26

Smith WHF, Sandwell DT (1997) Global seafloor topography from satellite altimetry and ship depth soundings: evidence for stochastic reheating of the oceanic lithosphere. Science 277:1956–1962

Smith WHF, Sandwell DT (2004) Conventional bathymetry, bathymetry from space, and geodetic altimetry. Oceanography 17:8–23

Song YT, Haidvogel DB (1994) A semi-implicit ocean circulation model using topography-following coordinate. Journal of Computational Physics 115:228–244

Song, YT, Hou TY (2006) Parametric vertical coordinate formulation for multiscale, Boussinesq, and non-Boussinesq ocean modeling. Ocean Model 11:298–332

Stammer D, Wunsch C, Ponte RM (2000) De-aliasing of global high frequency barotropic motions in altimeter observations. Geophys Res Lett 27:1175–1178

Sun S, Bleck R, Rooth CG, Dukowicz J, Chassignet EP, Killworth P (1999) Inclusion of thermobaricity in isopycnic-coordinate ocean models. J Phys Oceanogr 29:2719–2729

van Leer B (1977) Towards the ultimate conservative difference scheme IV: a new approach to numerical numerical convection. J Comput Phys 23:276–299

Veronis G (1975) The role of models in tracer studies. In: Numerical models of ocean circulation. National Academy of Sciences, Washington, pp. 133–146

Wallcraft AJ, Kara AB, Hurlburt HE, Rochford PA (2003) NRL Layered Ocean Model (NLOM) with an embedded mixed layer sub-model: formulation and tuning. J Atmos Oceanic Technol 20:1601–1615

Willebrand J, Barnier B, Böning C, Dieterich C, Killworth PD, Le Provost C, Jia Y, Molines JM, New AL (2001) Circulation characteristics in three eddy-permitting models of the North Atlantic. Prog Oceanogr 48:123–161

Xu X, ChangYS, Peters H, Özgökmen TM, Chassignet EP (2006). Parameterization of gravity current entrainment for ocean circulation models using a high-order 3D nonhydrostatic spectral element model. Ocean Model 14:19–44

Xu X, Chassignet EP, Price JF, Özgökmen TM, Peters H (2007) A regional modeling study of the entraining Mediterranean outflow. J Geophys Res 112:C12005. doi:10.1029/2007JC004145