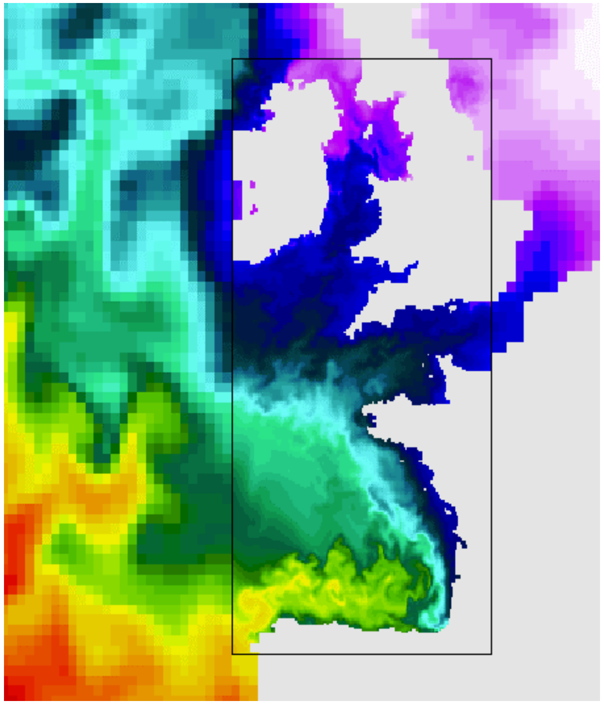
Nesting ocean models

E. Blayo
LMC-IMAG, University of Grenoble and INRIA Rhône-Alpes



Collaboration with :

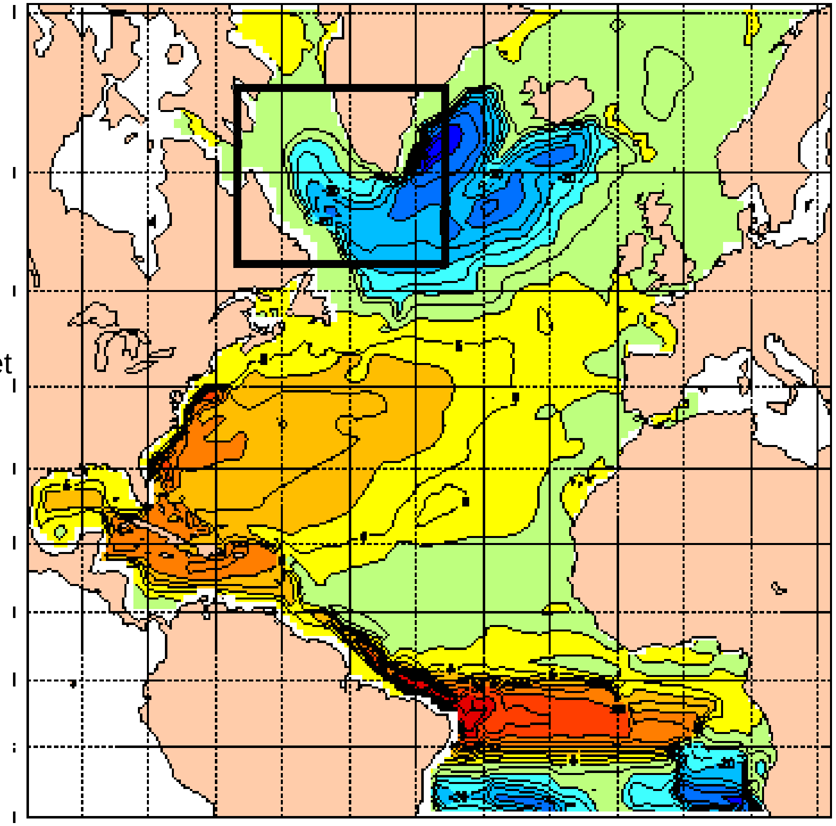
L.Debreu, V. Fedorenko, C. Vouland (LMC-IMAG)
B. Barnier, G. Broquet, S. Cailleau (LEGI, Grenoble)
L. Halpern, C. Japhet, V. Martin (LAGA Paris 13)
P. Marchesiello (IRD Brest)
F. Vandermeirsch (IFREMER Brest)

Mesh refinement in the Labrador sea
J. Chanut, L. Debreu (2002)

OPA model
+ AGRIF package

2 levels of resolution
(1/3° - 1/15°)

QuickTime™ et



cette image.

South of Brittany
M. Jouan, F. Dumas, L. Debreu (2003)

MARS model (IFREMER / SHOM),
+ AGRIF package

3 levels of resolution
(4.5 - 1.5 - 0.5 km)

Zoom method in an ancient coastal model
Asterix and Obelix (-51 B.C.)

The way a local model is forced at its open boundaries has a strong influence on the results.

- Try to classify the numerous problems and methods used in actual applications

- Discuss their theoretical validity and their practical use

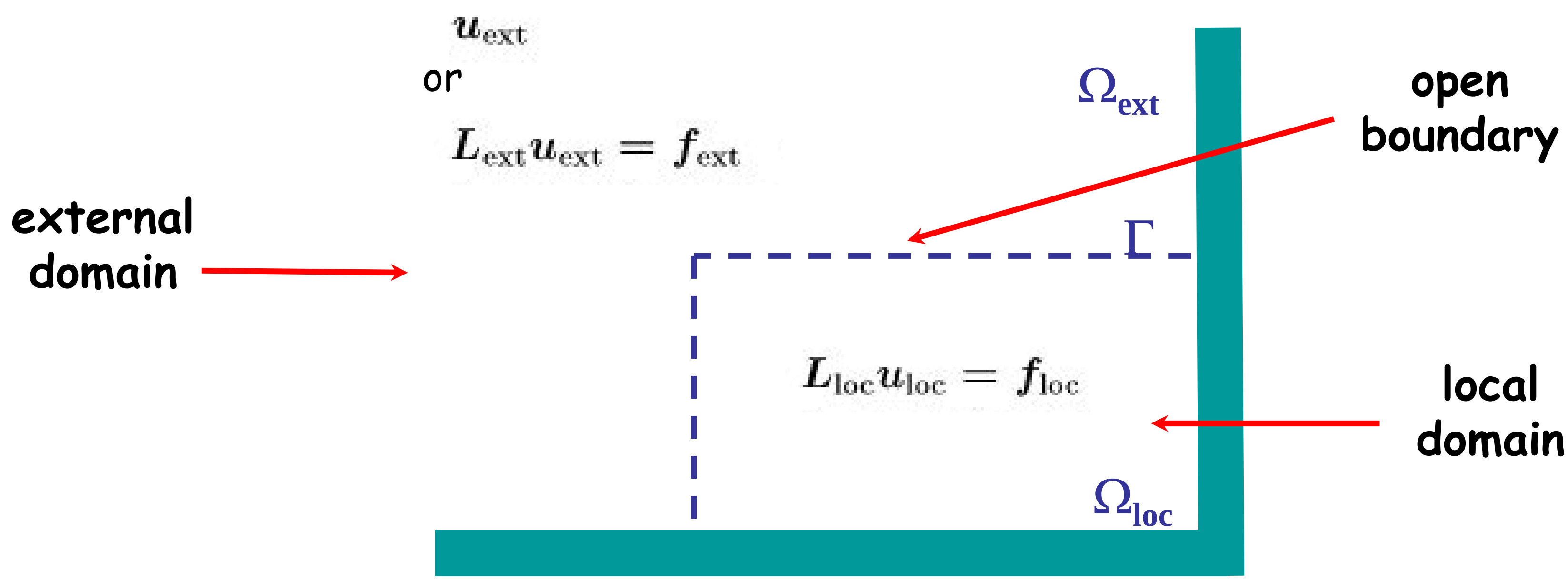
Outline :

1. A classification of nesting problems
2. The open boundary problem
3. Two way interactions
4. Software tools

Outline :

1. A classification of nesting problems
2. The open boundary problem
3. Two way interactions
4. Software tools

The nesting problem



We are mostly interested in the local solution.
The local model is more accurate than the external one.
Both models contain errors.

Which mathematical formulation(s) for this problem ?

How can we manage the artificial interface Γ ?

The inverse approach

Least-squares fit to the model and to the external solution (e.g. Bennett, 2002) :

Find u_{loc} that minimizes

$$\|L_{loc}u_{loc} - f_{loc}\|^2_{\Omega_{loc}\times[0,T]} \quad + \quad \varepsilon\,\|u_{loc} - u_{ext}\|^2_{\Gamma\times[0,T]}$$

or

Find (u_{loc}, u_{ext}) that minimizes

$$\|L_{loc}u_{loc} - f_{loc}\|^2_{\Omega_{loc}\times[0,T]} \quad + \quad \|L_{ext}u_{ext} - f_{ext}\|^2_{\Omega_{ext}\times[0,T]}$$
$$+ \quad \varepsilon\,\|u_{loc} - u_{ext}\|^2_{\Gamma\times[0,T]}$$

But :
- solving such a problem is difficult and expensive
- it requires a good choice of the norms, i.e. a good knowledge of the model and data errors

→ See lectures on data assimilation and inverse problems

The direct approach

Most actual nesting applications use a direct approach, i.e. the models equations are satisfied exactly (no least squares fit).

However, the exact formulation of the problem which is really solved is generally not expressed clearly, nor even considered. The nesting procedure is of algorithmic nature.

A correct formulation of the direct approach

Γ has no physical reality : transition as smooth as possible

A correct formulation is :

$$\text{Find } u_{loc} \text{ that satisfies}$$
$$\begin{cases} L_{loc}u_{loc} = f_{loc} & \text{in } \Omega_{loc} \times [0,T] \\ u_{loc} = u_{ext} \quad \text{and} \quad \dfrac{\partial u_{loc}}{\partial n} = \dfrac{\partial u_{ext}}{\partial n} & \text{on } \Gamma \times [0,T] \end{cases}$$
$$\text{under the constraint} \quad L_{ext}u_{ext} = f_{ext} \quad \text{in } \Omega_{ext} \times [0,T]$$

or equivalently :

$$\text{Find } u_{loc} \text{ and } u_{ext} \text{ that satisfy}$$
$$\begin{cases} L_{loc}u_{loc} = f_{loc} \quad \text{in } \Omega_{loc} \times [0,T] \quad \text{and} \quad L_{ext}u_{ext} = f_{ext} \quad \text{in } \Omega_{ext} \times [0,T] \\ \quad \text{with } u_{loc} = u_{ext} \quad \text{and} \quad \dfrac{\partial u_{loc}}{\partial n} = \dfrac{\partial u_{ext}}{\partial n} \quad \text{on } \Gamma \times [0,T] \end{cases}$$

A correct formulation of the direct approach (2)

Remark : L_{loc} and L_{ext} generally differ by several aspects, as well as f_{loc} and f_{ext} , or even Ω_{loc} and Ω_{ext}. To be able to get a smooth transition between u_{loc} and u_{ext} , ensure as far as possible a smooth transition between the models.

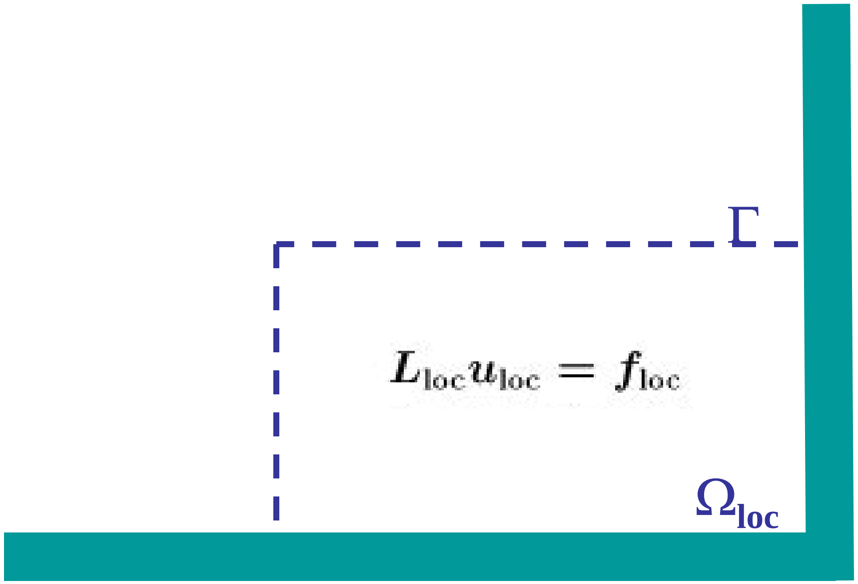
The usual approaches

- The external model is not always available for online interaction.
- The external model is generally not defined on Ω_{ext} only, but on $\Omega_{ext} + \Omega_{loc}$. Modifying it to avoid the overlap and implement an open boundary on Γ would be quite expensive.

➲ ***Actual applications do not address the correct problem but more or less approaching ones.***

The usual approaches (2)

The open boundary problem



$$\begin{cases} L_{loc} u_{loc} = f_{loc} & \text{in } \Omega_{loc} \times [0, T] \\ B u_{loc} = ?? & \text{on } \Gamma \times [0, T] \end{cases}$$

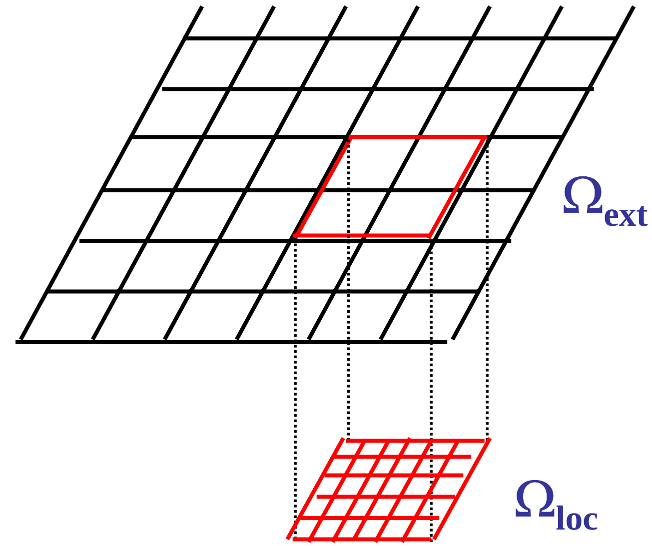
The usual approaches (3)

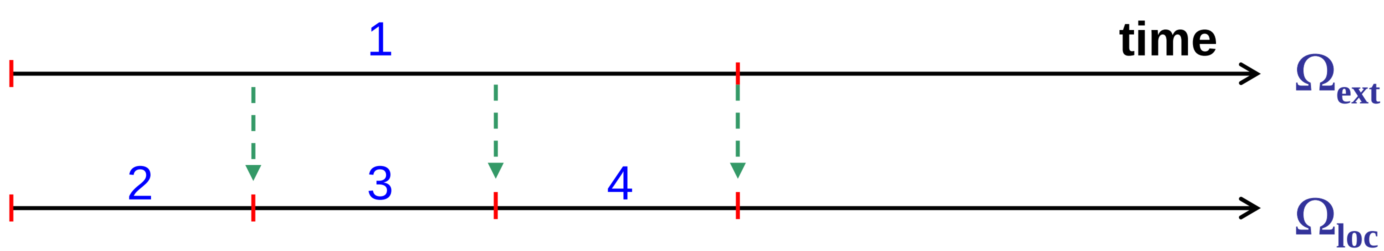
A particular case: one-way nesting

$$L_{ext}u_{ext} = f_{ext} \quad \text{in } \Omega_{ext} \cup \Omega_{loc} \times [0,T]$$

then

$$\begin{cases} L_{loc}u_{loc} = f_{loc} & \text{in } \Omega_{loc} \times [0,T] \\ Bu_{loc} = Bu_{ext} & \text{on } \Gamma \times [0,T] \end{cases}$$





- On-line / off-line interaction, subsampling of u_{ext}
- No feedback on u_{ext}

The usual approaches (4)

Usual two-way nesting

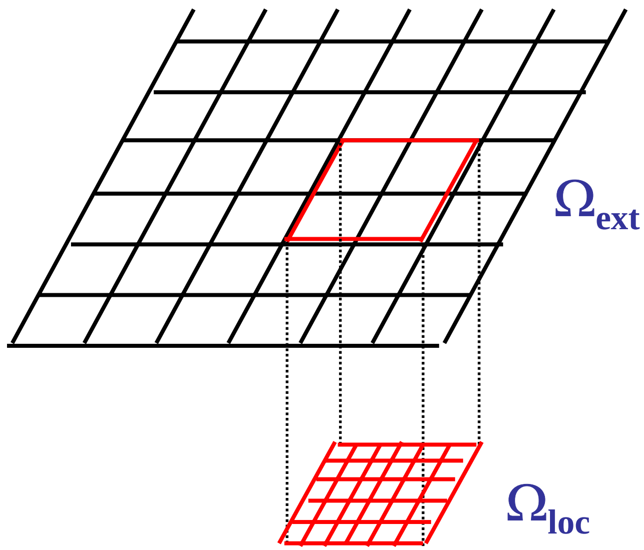
$$L_{ext}u_{ext} = f_{ext} \quad \text{in } \Omega_{ext} \cup \Omega_{loc} \times [0,T]$$

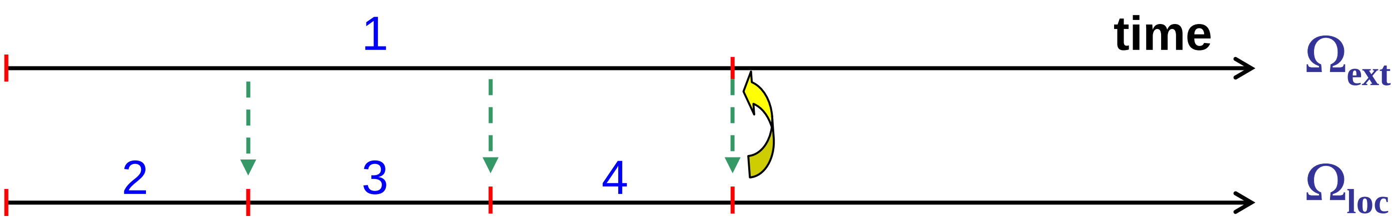
then

$$\begin{cases} L_{loc}u_{loc} = f_{loc} & \text{in } \Omega_{loc} \times [0,T] \\ Bu_{loc} = Bu_{ext} & \text{on } \Gamma \times [0,T] \end{cases}$$

then

$$u_{ext} = Hu_{loc} \quad \text{in } \Omega_{loc} \times [0,T]$$





- On-line interaction: the external model must be available
- Feedback on u_{ext}

The usual approaches (5)

The open boundary problem

$$\begin{cases} L_{loc} u_{loc} = f_{loc} & \text{in } \Omega_{loc} \times [0,T] \\ B u_{loc} = ?? & \text{on } \Gamma \times [0,T] \end{cases}$$

A particular case: one-way nesting

$$L_{ext} u_{ext} = f_{ext} \quad \text{in } \underline{\Omega_{ext} \cup \Omega_{loc}} \times [0,T]$$

then

$$\begin{cases} L_{loc} u_{loc} = f_{loc} & \text{in } \Omega_{loc} \times [0,T] \\ B u_{loc} = B u_{ext} & \text{on } \Gamma \times [0,T] \end{cases}$$

Usual two-way nesting

$$L_{ext} u_{ext} = f_{ext} \quad \text{in } \underline{\Omega_{ext} \cup \Omega_{loc}} \times [0,T]$$

then

$$\begin{cases} L_{loc} u_{loc} = f_{loc} & \text{in } \Omega_{loc} \times [0,T] \\ B u_{loc} = B u_{ext} & \text{on } \Gamma \times [0,T] \end{cases}$$

then

$$u_{ext} = H u_{loc} \quad \text{in } \Omega_{loc} \times [0,T]$$

Full coupling

$$L_{loc} u_{loc} = f_{loc} \quad \text{in } \Omega_{loc} \times [0,T]$$

and

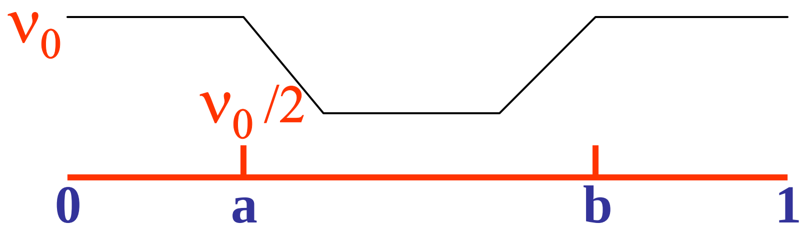
$$L_{ext} u_{ext} = f_{ext} \quad \text{in } \Omega_{ext} \times [0,T]$$

with $u_{loc} = u_{ext}$ on $\Gamma \times [0,T]$

and $\dfrac{\partial u_{loc}}{\partial n} = \dfrac{\partial u_{ext}}{\partial n}$ on $\Gamma \times [0,T]$

A 1-D numerical example

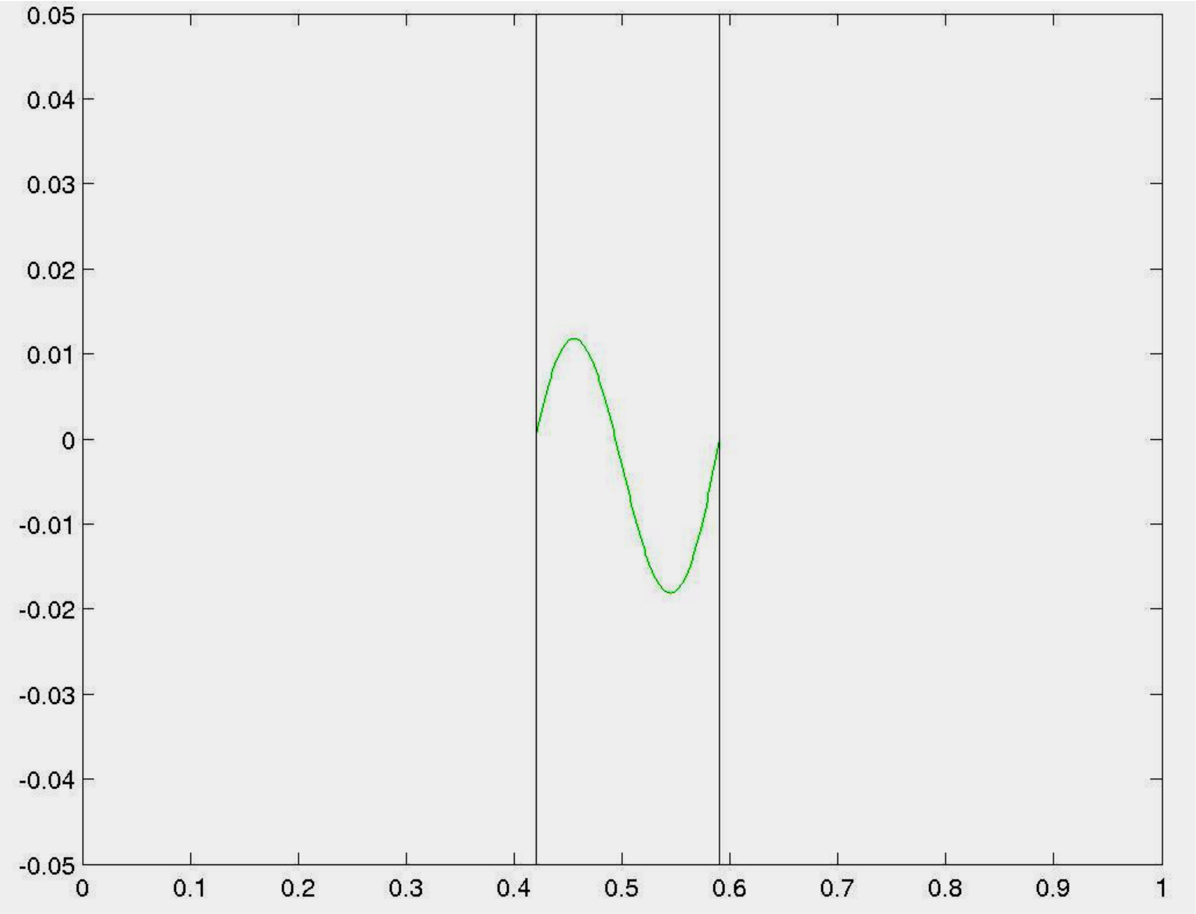
$$\begin{cases} -\nu(x)u''(x) + u(x) = \sin n\pi x & x \in]0,1[\\ u(0) = u(1) = 0 \end{cases}$$



A 1-D numerical example (2)

Open boundary approach :

$$\begin{cases} -\nu(x)u''(x) + u(x) = \sin n\pi x & x \in]a, b[\\ u(a) = \alpha \qquad u(b) = \beta & \end{cases}$$



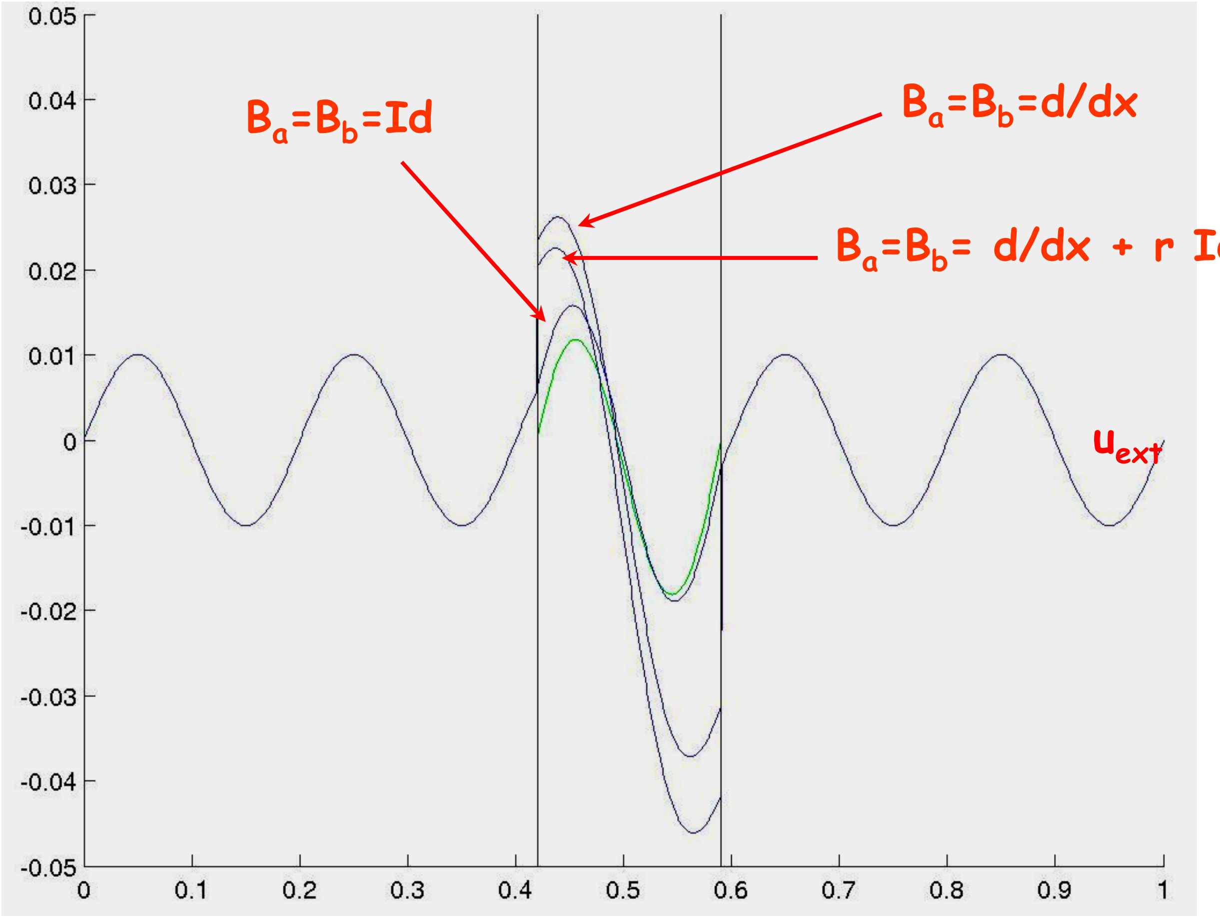
A 1-D numerical example (3)

One-way/two-way nesting :

$$\begin{cases} -\nu_0\, u''_{ext}(x) + u_{ext}(x) = \sin n\pi x, & x \in]0,1[\\ u_{ext}(0) = u_{ext}(1) = 0 \end{cases}$$

then

$$\begin{cases} -\nu(x) u''_{nes}(x) + u_{nes}(x) = \sin n\pi x, & x \in]a,b[\\ B_a u_{nes}(a) = B_a u_{ext}(a) \quad \text{and} \quad B_b u_{nes}(b) = B_b u_{ext}(b) \end{cases}$$

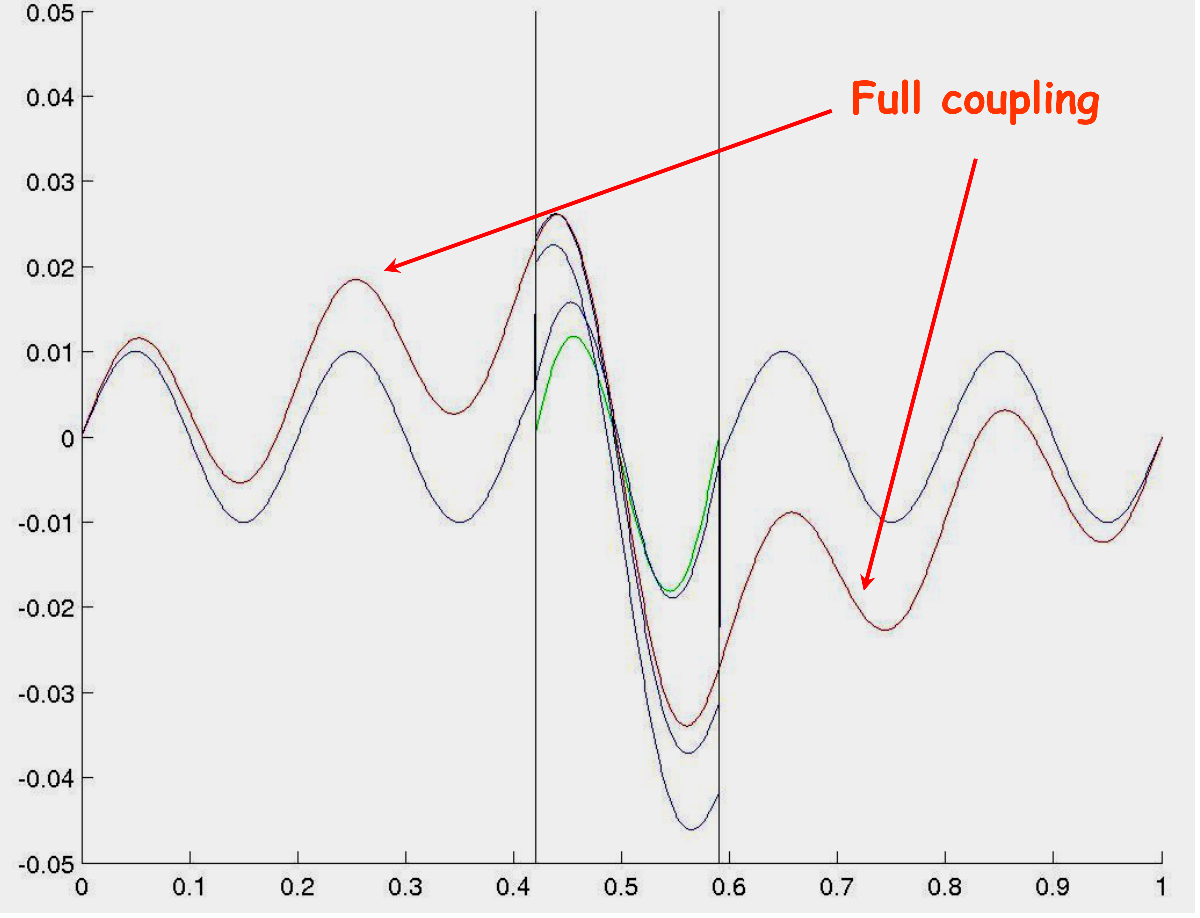


A 1-D numerical example (4)

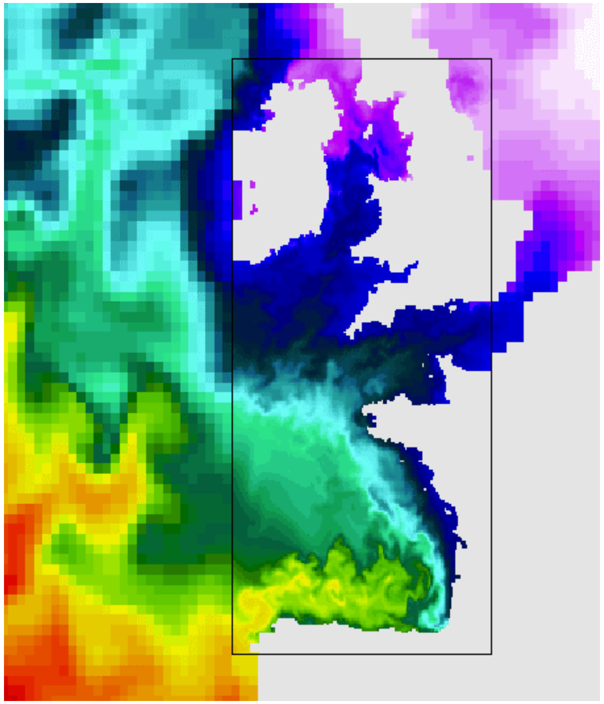
Full coupling :

$$\begin{cases} -\nu(x)u''(x) + u(x) = \sin n\pi x & x \in]0,1[\\ u(0) = u(1) = 0 \end{cases}$$

This problem admits a unique solution, which is continuous and derivable.



A realistic example



Bay of Biscay 1/15°

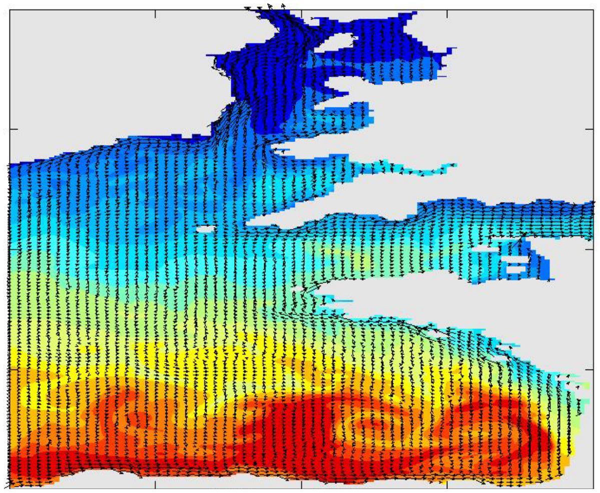
-

North Atlantic 1/3°

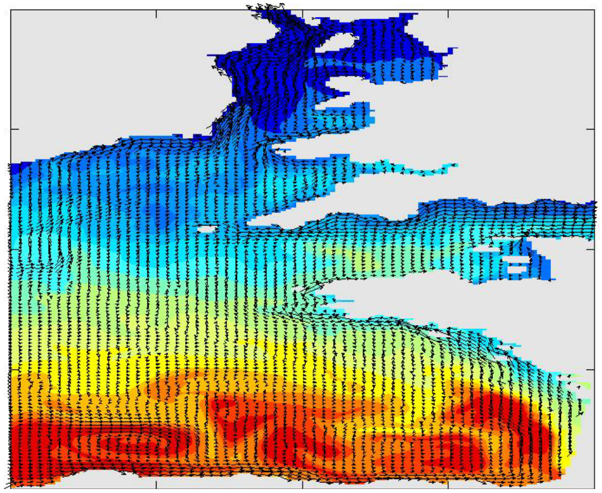
Cailleau and Fedorenko, 2004

Surface temperature on dec.29, 1998

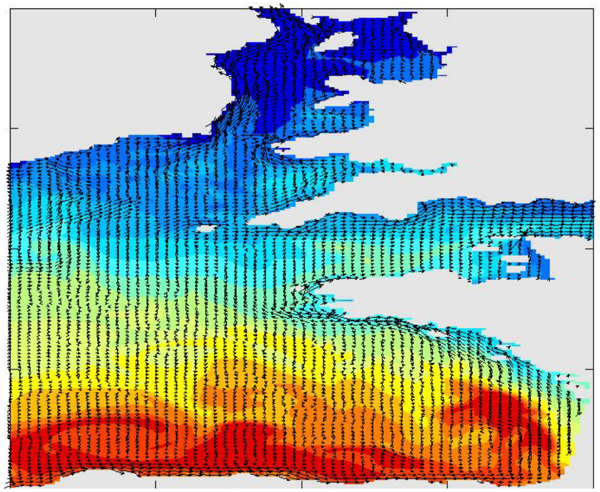
One-way



Two-way

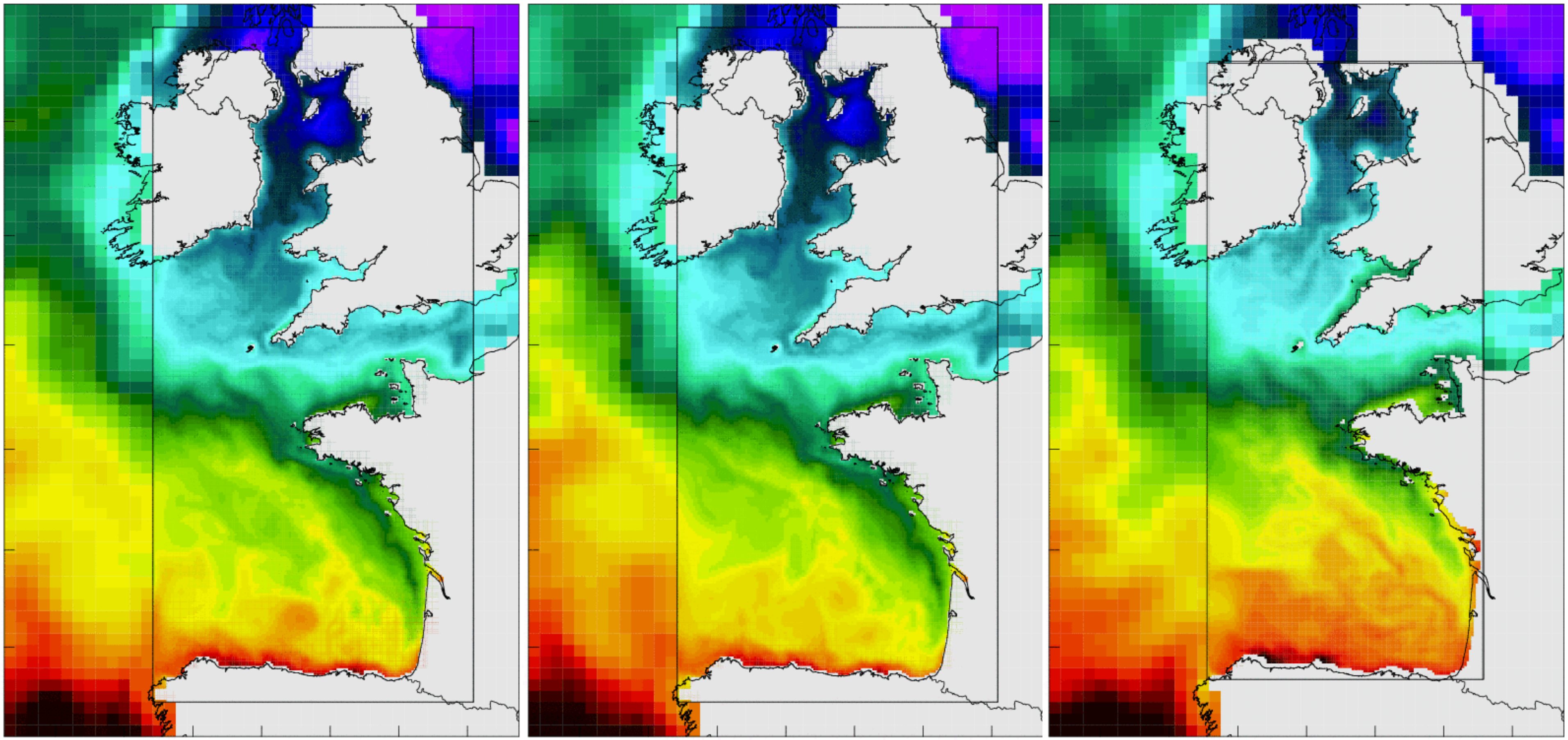


Full coupling



A realistic example (2)

One-way | Two-way | Full coupling

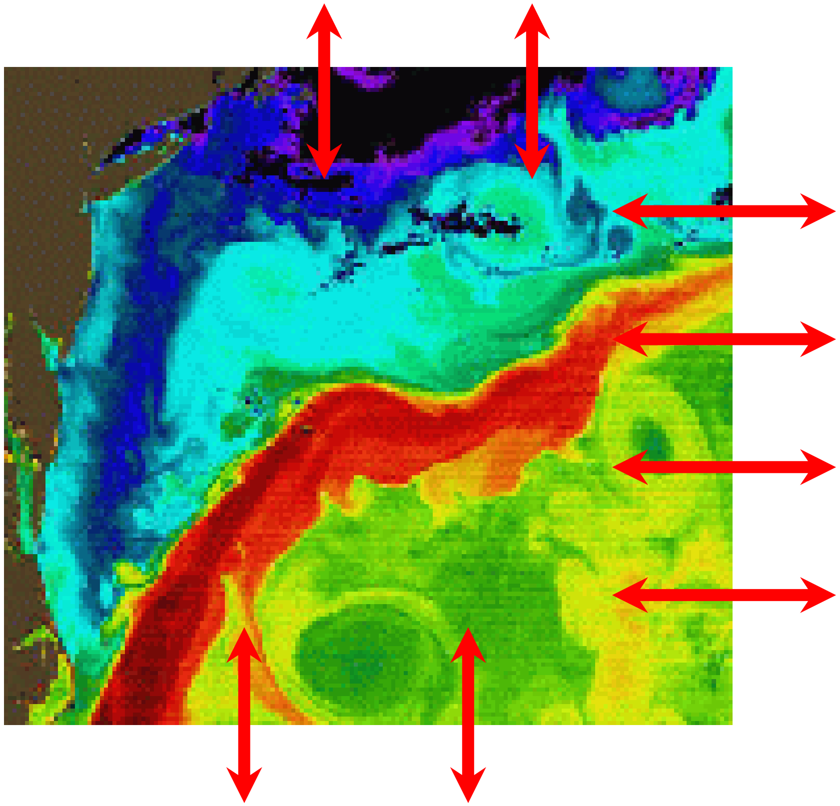


3-months average (april-may-june 1998) - temperature at z=30m

Outline :

1. A classification of nesting problems
2. **The open boundary problem**
3. Two way interactions
4. Software tools

The open boundary problem / one-way nesting



u^{ext}

v^{ext} (climatological data, large-scale model…)

T^{ext}

…

$$\begin{cases} L_{loc} u_{loc} = f_{loc} & \text{in } \Omega_{loc} \times [0,T] \\ B u_{loc} = ?? & \text{on } \Gamma \times [0,T] \end{cases}$$

Goal : choose the partial differential operator B in order to

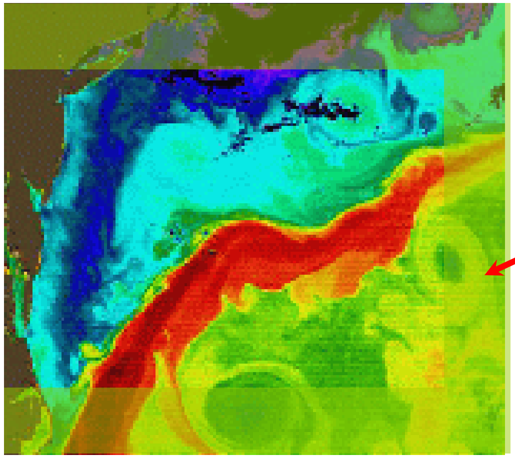
- evacuate the outgoing information
- bring some external knowledge on incoming information

The open boundary problem (2)

- OBCs (data + operator) have a strong influence on the model results.
- Designing OBCs is an old problem : abundant literature, numerous conditions proposed, often no clear conclusions.
- Some OBCs are often recommended in comparative studies : radiation methods with relaxation term, Flather condition, relaxation (+ sponge layer), Hedström condition.

  - give an overview of usual OBCs
  - performances of OBCs : fact and fiction
  - How can we design OBCs for primitive equations ?

Usual open boundary conditions : relaxation methods



relaxation layer

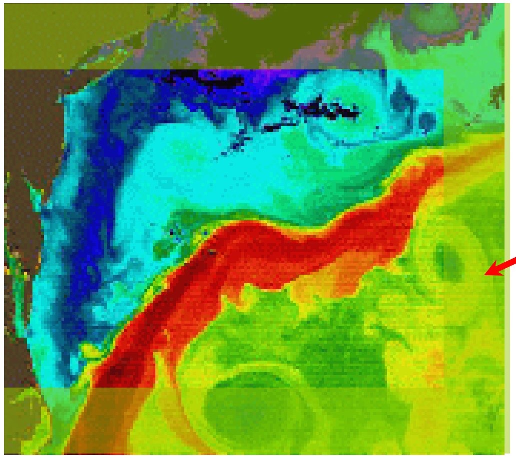
$$(1-\alpha(x))\,\phi+\alpha(x)\,\phi^{\text{ext}}$$

- This is equivalent to adding a nudging term: $\frac{\partial \phi}{\partial t}+F(\phi)+K(\phi-\phi^{\text{ext}})=0$

- Relaxation is often used together with a sponge layer (i.e. increased viscosity within the relaxation area)

- Limit case : Clamped boundary conditions $\phi=\phi^{\text{ext}}$

Usual open boundary conditions : relaxation methods (2)



$$(1-\alpha(x))\,\phi + \alpha(x)\,\phi^{\text{ext}}$$

- Clamped boundary conditions $\phi = \phi^{\text{ext}}$ should be avoided : the outgoing information does not depend on the interior solution.

- Relaxation + sponge layer gives good results in all comparative studies (e.g. Röed & Cooper, 87; Palma & Matano, 98; Nycander & Döös, 03).

- Drawbacks: - increase of computational cost
  - solution in the relaxation layer ?

- Future methods : Perfectly Matched Layers (Berenger, 1994; Hu, 2001; Navon et al, 2004)

Usual open boundary conditions : radiation methods

- Radiation conditions are based on the Sommerfeld condition :

$$\frac{\partial \phi}{\partial t} + c\frac{\partial \phi}{\partial x} = 0$$

- For complex flows: adaptive evaluation of c (Orlanski-type methods)

c = c(x,t) is computed at each gridpoint on the boundary and at each timestep, using inner values at the same or at previous timesteps.

$$c(x,t) = -\frac{\partial \phi / \partial t}{\partial \phi / \partial x}$$ gives for example $$c_B^n = -\frac{\Delta x}{\Delta t}\frac{\phi_{B-1}^{n-1} - \phi_{B-1}^{n-2}}{\phi_{B-1}^{n-1} - \phi_{B-2}^{n-1}}$$

Numerous variants : Orlanski (76), Camerlengo & O'Brien (80), Miller & Thorpe (81), Raymond & Kuo (84), Barnier et al. (98), Marchesiello et al. (01)

- Results and recommendations are split.

Usual open boundary conditions : radiation methods (2)

Orlanski = Sommerfeld + an arbitrary hypothesis e.g. $\frac{\partial c}{\partial x} = 0$

- This additional hypothesis makes the condition nonlinear (Nycander & Döös, 03).

$$\frac{\partial^2 \phi}{\partial t \partial x}\frac{\partial \phi}{\partial x} - \frac{\partial \phi}{\partial t}\frac{\partial^2 \phi}{\partial x^2} = 0$$

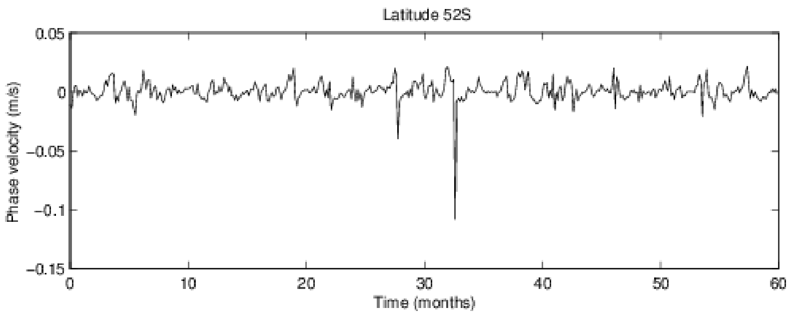
- Orlanski-type methods have no justification as soon as there is a combination of several waves with different velocities :

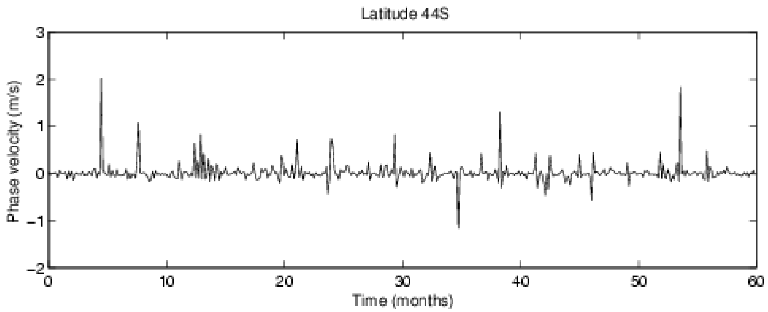
$F(x,t)=f(x-ct)$ satisfies any Orlanski-type condition,

but $F(x,t)=f_1(x-c_1t)+f_2(x-c_2t)$ with $c_1 <> c_2$ does not.

Usual open boundary conditions : radiation methods (3)

➢ The adaptive estimation of *c* has no physical meaning, but is more or less a white noise (Tréguier et al., 2001; Durran, 2001).





➢ Their apparent efficiency in some studies is mostly due to the addition of a relaxation term.

$$\begin{cases} \phi = \phi^{\text{ext}} \quad \text{or} \quad \dfrac{\partial \phi}{\partial t} = -\dfrac{\phi - \phi^{\text{ext}}}{\tau_{\text{in}}} & \text{if } c \text{ is inward} \\ \dfrac{\partial \phi}{\partial t} + c\dfrac{\partial \phi}{\partial n} = -\dfrac{\phi - \phi^{\text{ext}}}{\tau_{\text{out}}} & \text{if } c \text{ is outward} \end{cases}$$

$$\frac{\partial(\phi - \phi^{\text{ext}})}{\partial t} = -\frac{\phi - \phi^{\text{ext}}}{\tau} \quad \text{or} \quad \phi = \phi^{\text{ext}}$$ would give quite similar results.

Usual open boundary conditions : Flather condition

For 2-D barotropic flows :

Sommerfeld condition for surface elevation : $\frac{\partial \eta}{\partial t} + \sqrt{gh}\,\frac{\partial \eta}{\partial n} = 0$

1-D approximation of the continuity equation : $\frac{\partial \eta}{\partial t} + h\,\frac{\partial \bar{v}_n}{\partial n} = 0$

Combination + integration through Γ : $\bar{v}_n - \sqrt{\frac{g}{h}}\,\eta = \bar{v}_n^{ext} - \sqrt{\frac{g}{h}}\,\eta^{ext}$

- Used with additional conditions like $\frac{\partial h}{\partial n} = 0$
- Gives good results in all comparative studies (e.g. Palma & Matano, 98; Marchesiello et al, 01; Nycander & Döös, 03).

Usual open boundary conditions : model adapted methods

Relaxation and radiation methods do not depend of the model equations. Other methods provide OBCs adapted to the system :

- Characteristic waves amplitudes (Hedström) methods (e.g. Bruneau and Creusé, 01)
- Absorbing conditions : local approximation of exact conditions (Engquist and Majda, 77)

➢ Much more complicated to handle: restricted presently to simple 1-D or 2-D models

➢ Several recent successful applications in such simple models (Mc Donald, 02; Nycander & Döös, 03; Martin, 03)

Usual open boundary conditions

How can we discriminate these numerous OBCs ? Are they justified or not ? Can we explain their performances ?

→ Two important criteria (Blayo and Debreu, 2004)

Criterion #1 : working on characteristic variables

Ocean models are basically hyperbolic systems of nonlinear equations, with the addition of viscous terms and degenerative hypotheses (e.g. hydrostatic assumption).

$$\frac{\partial \Phi}{\partial t} + A(\Phi)\,\frac{\partial \Phi}{\partial x} = F$$

Diagonalizing *A* allows a decomposition into incoming and outgoing characteristic variables (or Riemann invariants).

- 1 OBC needed for each incoming variable
- no OBC for outgoing variables (upwind schemes or extrapolation)

Criterion #1 : working on characteristic variables (2)

Example : shallow-water equations - eastern open boundary

$$\begin{cases} \dfrac{\partial u}{\partial t} + u_0 \dfrac{\partial u}{\partial x} + v_0 \dfrac{\partial u}{\partial y} + g \dfrac{\partial h}{\partial x} - f v = F_x \\[2ex] \dfrac{\partial v}{\partial t} + u_0 \dfrac{\partial v}{\partial x} + v_0 \dfrac{\partial v}{\partial y} + g \dfrac{\partial h}{\partial y} + f u = F_y \\[2ex] \dfrac{\partial h}{\partial t} + u_0 \dfrac{\partial h}{\partial x} + v_0 \dfrac{\partial h}{\partial y} + h_0 \left(\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} \right) = 0 \end{cases}$$

i.e. $\dfrac{\partial \Phi}{\partial t} + A_1 \dfrac{\partial \Phi}{\partial x} + A_2 \dfrac{\partial \Phi}{\partial y} + B\Phi = F$ with $\Phi = \begin{pmatrix} u \\ v \\ h \end{pmatrix}$ $A_1 = \begin{pmatrix} u_0 & 0 & g \\ 0 & u_0 & 0 \\ h_0 & 0 & u_0 \end{pmatrix}$

Diagonalizing A_1 gives the characteristic variables :

$$w_1 = u - \sqrt{\frac{g}{h_0}}\, h \qquad w_2 = v \qquad w_3 = u + \sqrt{\frac{g}{h_0}}\, h$$

Criterion #1 : working on characteristic variables (3)

$$\begin{cases} \dfrac{\partial w_1}{\partial t} + (u_0 - c)\dfrac{\partial w_1}{\partial x} + v_0\dfrac{\partial w_1}{\partial y} - c\dfrac{\partial v}{\partial y} - fv = F_x \\ \dfrac{\partial v}{\partial t} + u_0\dfrac{\partial v}{\partial x} + v_0\dfrac{\partial v}{\partial y} + \dfrac{c}{2}\dfrac{\partial (w_3 - w_1)}{\partial y} + \dfrac{f}{2}(w_1 + w_3) = F_y \\ \dfrac{\partial w_3}{\partial t} + (u_0 + c)\dfrac{\partial w_3}{\partial x} + v_0\dfrac{\partial w_3}{\partial y} + c\dfrac{\partial v}{\partial y} - fv = F_x \end{cases}$$

with $c = \sqrt{gh_0}$

In case of an eastern open boundary:

1 OBC for w_1 (always entering) + 1 OBC for v when $u_0<0$

Criterion #2 : consistency with external data

OBCs connect the model solution to external data.

For this connection to be mathematically regular : $B\phi = B\phi^{ext}$

- Simplest choices : $B = Id$ or $B = \frac{\partial}{\partial n}$
- Ensures asymptotic consistency (Engquist and Halpern, 88)

- **$Bw = Bw^{ext}$ for each incoming characteristic variable w**
- **These two criteria explain the behavior and performances of usual OBCs.**

Revisiting usual OBCs using these criteria : Flather condition

- This condition satisfies the two preceding criteria :

$$u - \sqrt{\frac{g}{h_0}}\, h = u^{\text{ext}} - \sqrt{\frac{g}{h_0}}\, h^{\text{ext}}$$

i.e. $w_1 = w_1^{ext}$

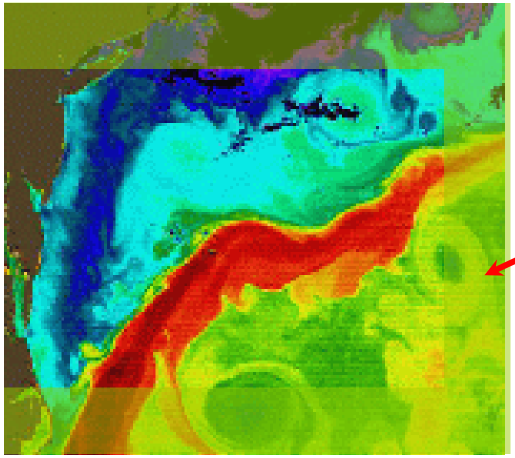
Revisiting usual OBCs using these criteria : model adapted methods

➢ Absorbing conditions work on characteristic variables

First order absorbing conditions : $w = w^{ext}$ for each incoming w

➢ Characteristic waves amplitudes methods : work on **dw/dn** for each incoming w

Revisiting usual OBCs using these criteria : relaxation methods



$$(1 - \alpha(x))\,\phi + \alpha(x)\,\phi^{\text{ext}}$$

- The treatment of characteristic variables is implicit.
- This is not the case for clamped conditions.

Revisiting usual OBCs using these criteria : radiation methods

- Radiation conditions are based on the Sommerfeld condition :

$$\frac{\partial \phi}{\partial t} + c\,\frac{\partial \phi}{\partial x} = 0$$

- This condition is fully justified for single wave propagation problems.

$$\frac{\partial^2 \phi}{\partial t^2} - c^2\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right) = 0$$ can be rewritten $$\frac{\partial \Phi}{\partial t} + A_1\frac{\partial \Phi}{\partial x} + A_2\frac{\partial \Phi}{\partial y} = 0$$

with $$\Phi = \begin{pmatrix} \partial\phi/\partial t \\ \partial\phi/\partial x \\ \partial\phi/\partial y \end{pmatrix}, \quad A_1 = \begin{pmatrix} 0 & -c^2 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 0 & 0 & -c^2 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}.$$

Diagonalizing A_1 gives the incoming characteristic : $w = \frac{\partial \phi}{\partial t} + c\,\frac{\partial \phi}{\partial x}$

This is the origin of the Sommerfeld condition. It has not to be used out of this context of wave equations.

Towards characteristic based OBCs for primitive equations

How could these criteria be applied to design OBCs for primitive equations ?

Definition of a characteristic-based OBC :

At each location on the open boundary and at each timestep :

- consider the hyperbolic part of the locally linearized equations
- determine incoming and outgoing characteristic variables
- for each incoming characteristic variable w, impose an OBC of the form : $Bw = Bw^{ext}$
- for each outgoing characteristic variable, compute its value on the open boundary from interior values using upwind schemes or extrapolation

Towards characteristic-based OBCs for primitive equations (2)

shallow-water system

Characteristic variables :

$$w_1 = u - \sqrt{\frac{g}{h_0}}\,h \quad (u-c) \qquad w_2 = v \quad (u) \qquad w_3 = u + \sqrt{\frac{g}{h_0}}\,h \quad (u+c)$$

- OBC :

  if $u>0$ (outgoing flow) : $B_1\, w_1 = B_1\, w_1^{ext}$

  if $u<0$ (incoming flow) : $B_2\, w_1 = B_2\, w_1^{ext}$ and $B_3\, v = B_3\, v^{ext}$

- Simplest choices : $B = Id$ or $B = \frac{\partial}{\partial n}$ ($B_1 = B_2 = Id$: Flather)
- No need to add arbitrary OBCs like $\frac{\partial h}{\partial n} = 0$ for instance.
- Good behaviour in numerical experiments

Towards characteristic-based OBCs for primitive equations (3)

- **Barotropic dynamics :** shallow-water system

- **Baroclinic dynamics :** loss of hyperbolicity, due to the hydrostatic assumption. A possibility could be to use a decomposition into vertical modes (under investigation). Relaxation is another possible method.

- **Tracers :**

$$\frac{\partial T}{\partial t} + \mathrm{U}.\nabla T - \nabla.(A_T \nabla T) = F_T$$

Hyperbolic part : $\frac{\partial T}{\partial t} + \mathrm{U}.\nabla T = 0$

Characteristic variable : *T* (associated with eigenvalue *U.n*)

$B\,T = B\,T^{ext}$ if **U.n<0** (no justification for using Orlanski-type OBCs)

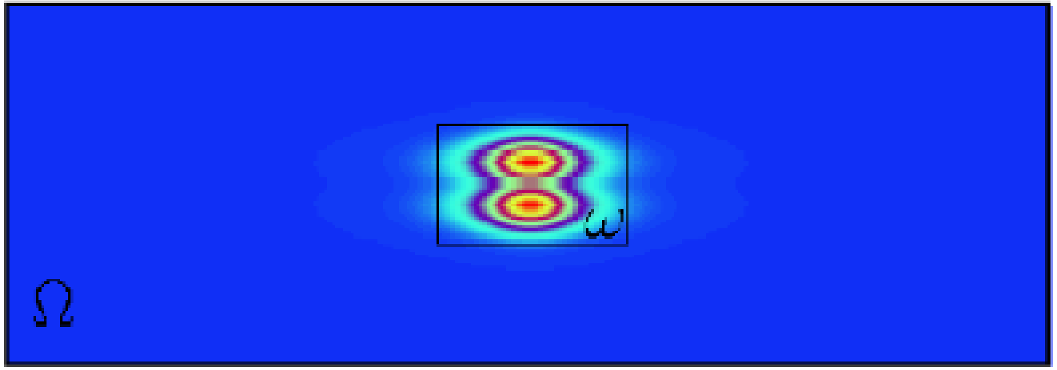
Some practical remarks

Numerical modes We discuss here only the continuous form of the equations. Numerical models contain spurious modes, which must be handled by the OBCs --> additional specific work.

Quality of the external data Incoming information is entirely dependent of the external data. The quality of these data is an important aspect for the performance of a regional modelling system.

Initialization problem The initial condition is generally built by interpolation of a larger scale solution, which is not perfectly consistent with the local model. This can lead to a long adjustment phase.
--> add some relevant constraints in the computation of the initial condition (e.g. Auclair et al., 2000).

Several OBCs will be illustrated during the exercise on tomorrow afternoon.



Outline :

1. A classification of nesting problems
2. The open boundary problem
3. **Two way interactions**
4. Software tools

Usual two-way nesting

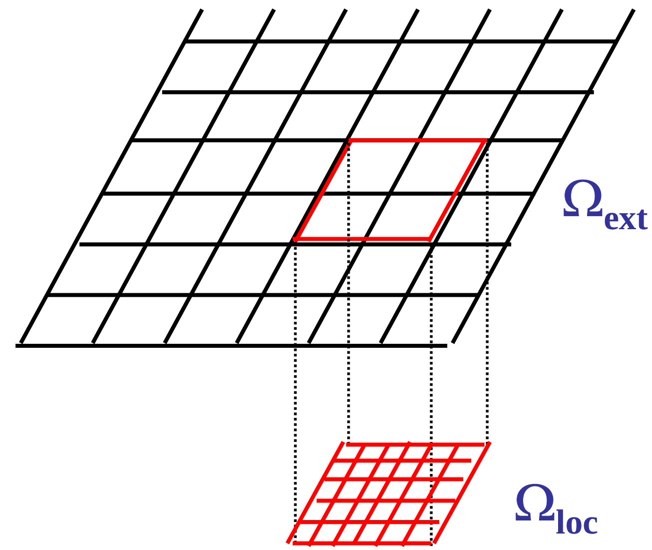
$$L_{ext}u_{ext} = f_{ext} \quad \text{in } \Omega_{ext} \cup \Omega_{loc} \times [0,T]$$

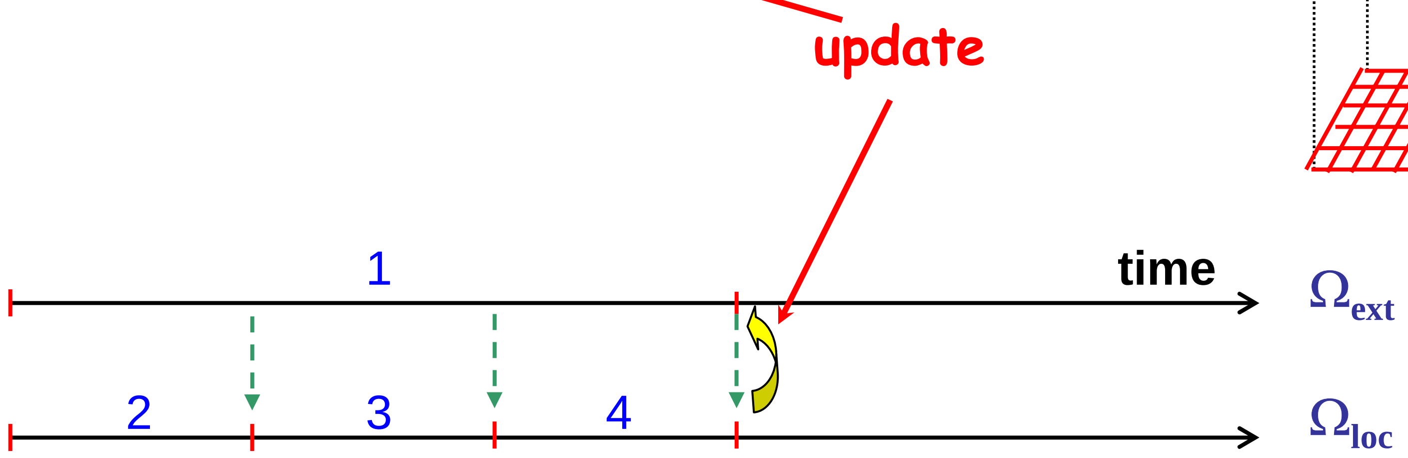
then

$$\begin{cases} L_{loc}u_{loc} = f_{loc} & \text{in } \Omega_{loc} \times [0,T] \\ Bu_{loc} = Bu_{ext} & \text{on } \Gamma \times [0,T] \end{cases}$$

then

$$u_{ext} = Hu_{loc} \quad \text{in } \Omega_{loc} \times [0,T]$$



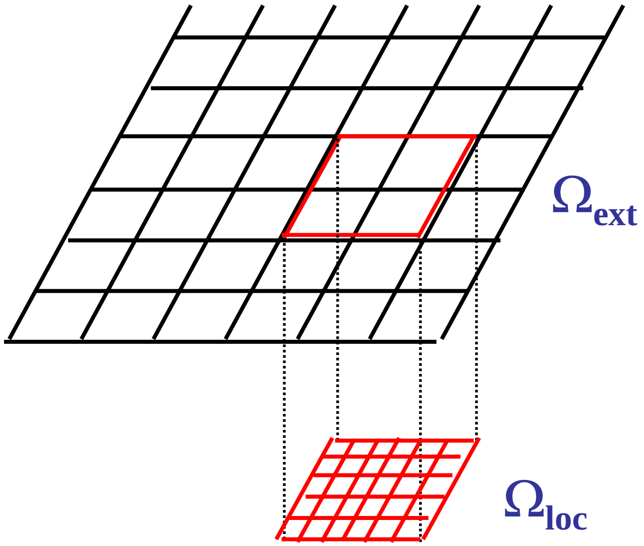


- On-line interaction: the external model must be available
- Feedback on u_{ext} to improve both solutions

Usual two-way nesting (2)

- Update operator : copy, average…

- Possibly : additional flux correction step, to ensure balance of mass, of tracer fluxes… u_{loc} --> u^*_{loc}



- Two-way interaction generally decreases the difficulties encountered in one-way interaction, especially along Γ, and seems to improve the model solution.

- However the connection between u_{loc} and u_{ext} is still unsmooth :
before the flux correction step : continuity, but unbalanced fluxes
after the flux correction step : balanced fluxes but discontinuity

Full coupling: Schwarz methods

Obtaining a solution of the original problem :

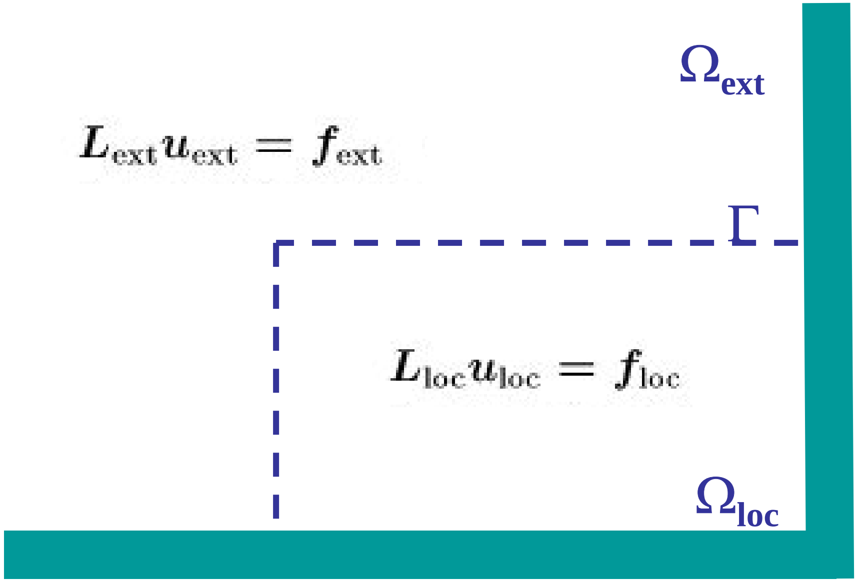
Find u_{loc} and u_{ext} that satisfy

$$\begin{cases} L_{loc}u_{loc} = f_{loc} \quad \text{in } \Omega_{loc} \times [0,T] \quad \text{and} \quad L_{ext}u_{ext} = f_{ext} \quad \text{in } \Omega_{ext} \times [0,T] \\ \quad \text{with } u_{loc} = u_{ext} \quad \text{and} \quad \dfrac{\partial u_{loc}}{\partial n} = \dfrac{\partial u_{ext}}{\partial n} \quad \text{on } \Gamma \times [0,T] \end{cases}$$

is much more difficult and expensive than the previous algorithms.

- Modify the external model to suppress the overlap and to implement an open boundary on Γ.

- Implement an algorithm to solve the problem.



→ Never been addressed before in ocean/atmosphere modelling

Full coupling: Schwarz methods (2)

General framework of domain decomposition methods

Global-in-time non-overlapping Schwarz algorithms :

$$\left\{ \begin{array}{lll} L_{loc} u_{loc}^{n+1} & = f_{loc} & \text{in } \Omega_{loc} \times [0,T] \\ u_{loc}^{n+1} & \text{given} & \text{at } t=0 \\ B_{loc} u_{loc}^{n+1} & = B_{loc} u_{ext}^{n} & \text{on } \Gamma \times [0,T] \end{array} \right. \quad \text{and} \quad \left\{ \begin{array}{lll} L_{ext} u_{ext}^{n+1} & = f_{ext} & \text{in } \Omega_{ext} \times [0,T] \\ u_{ext}^{n+1} & \text{given} & \text{at } t=0 \\ B_{ext} u_{ext}^{n+1} & = B_{ext} u_{loc}^{n} & \text{on } \Gamma \times [0,T] \end{array} \right.$$

- Iterative methods : the cost is multiplied by the number of iterations.
- The convergence rate depends on the choice of B_{loc} and B_{ext}.

Issue : find efficient operators (close links with absorbing conditions).

- On-going research work.

Outline :

1. A classification of nesting problems
2. The open boundary problem
3. Two way interactions
4. **Software tools**

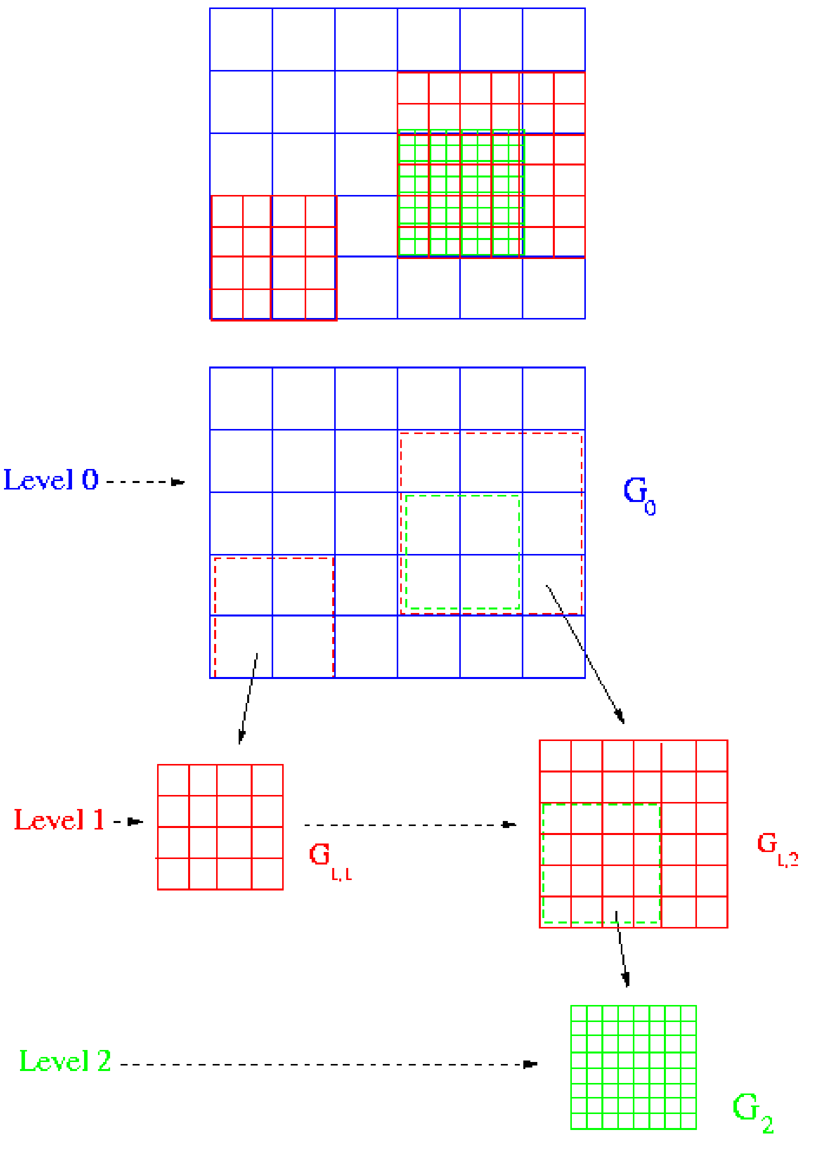
Software tools for nesting and coupling

Designing nested or coupled systems is quite a difficult and time consuming practical task. Several softwares have been developed, which automatically manage an important part of the job.

Software tools for nesting and coupling (2)

Mesh refinement

- local and external models are different versions of the same source code
- matching grids (fixed grid ratio)



Agrif package : Allows an easy integration of mesh refinement capabilities within any existing finite difference Fortran model.
One-way / two-way (adaptive) multiply-nested systems. (Debreu et al., 04)
http://www-lmc.imag.fr/IDOPT/AGRIF

Software tools for nesting and coupling (3)

Coupling tools

- local and external models are different source codes
- non-matching grids

The couplers automaticallly manage a number of programming details : synchronizations, transfers of data…

PALM : http://www.cerfacs.fr/~palm
MpCCI : http://www.mpcci.org