Finite-Element Ocean circulation Model (FEOM)

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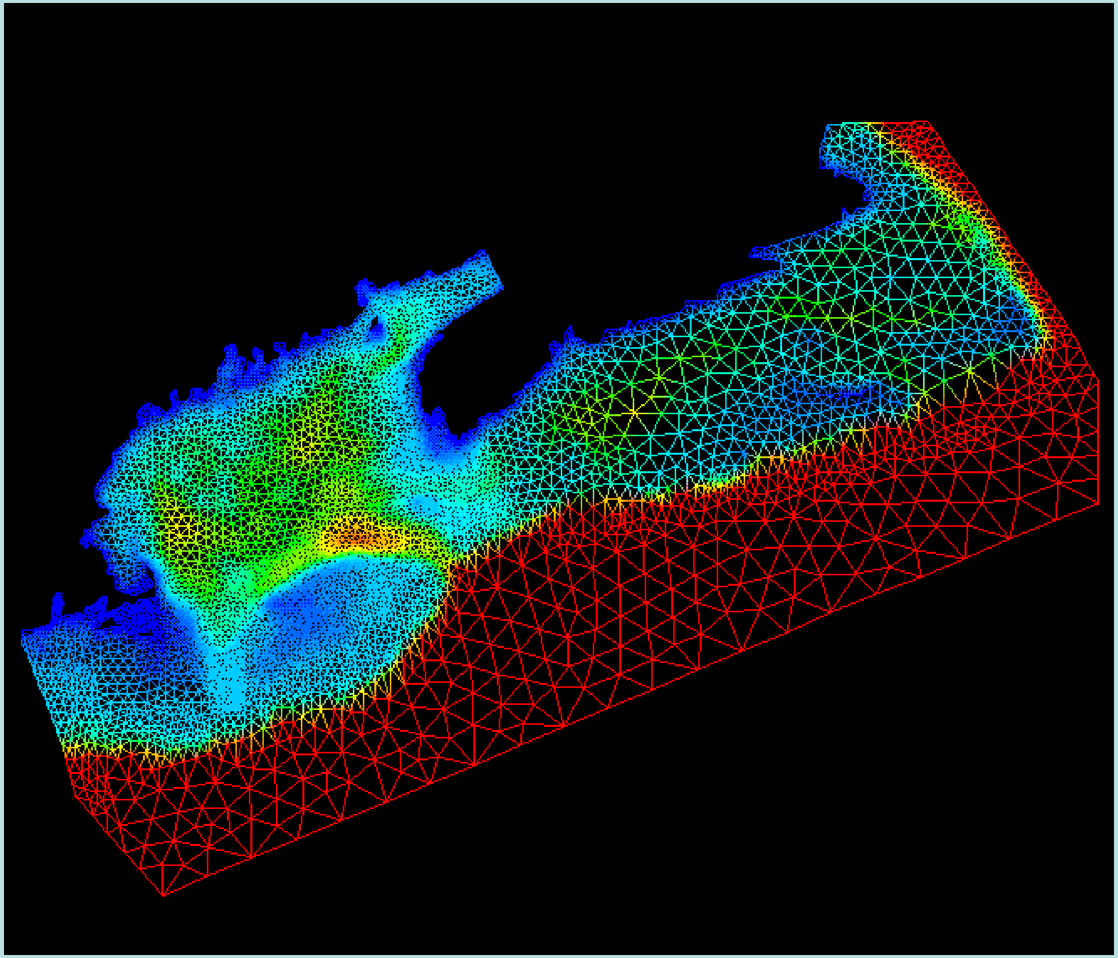
Motivation:

- Complexity of coastal lines
- Need for very high resolution in dynamically important regions
- Sloping bottom topography

Finite-element discretization provides a framework

Long history of using FE in tidal and coastal modeling (QUODDY, ADCIRC) :



QUODDY mesh for the Gulf of Maine, the spatial resolution varies from 4 to 50 km

Main interest - surface elevation

Main forcing – wind and tides

$\Downarrow$

Spatial scales $\propto \sqrt{H}$

Typical integration – a few months

FE (unstructured) models

• tides:	FESxx, Mog2D etc.
• shallow water:	Untrimm, etc.
• coastal:	Quoddy, Adcirc, Ricom FVcom, SEOM, Elcirc…
• engeneering	Delft
• convection etc.	ICOM
• basin scale	RAS, FEOM
• atmosphere	ICON, Canda

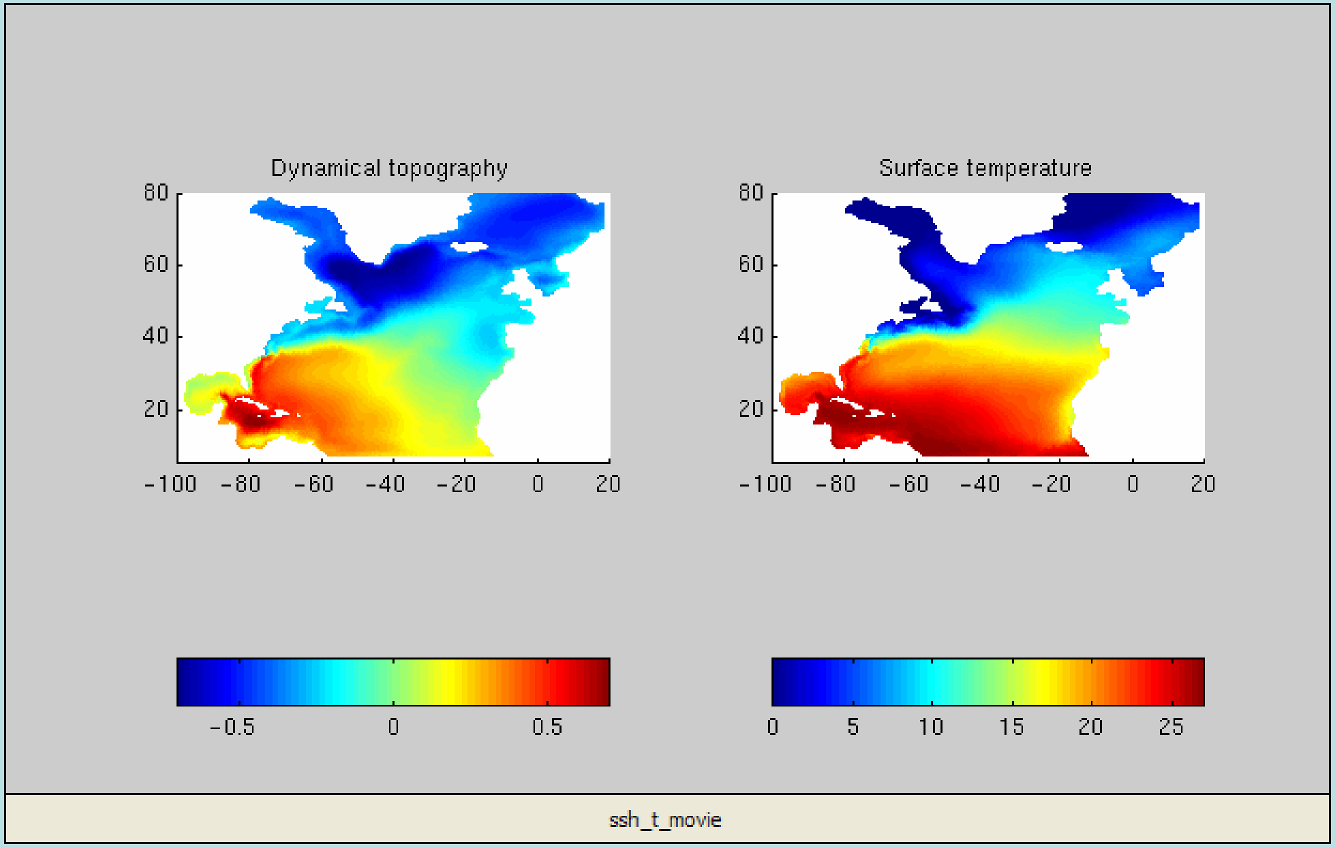
Ocean GCM traditionally use Finite Difference approach

"... there are two general problems which have arisen when attempting to use unstructured grid in climate models.
The first is that it is difficult to represent the geostrophic balance correctly. ...
The second is that every change in grid spacing provides an opportunity for unphysical wave scattering....

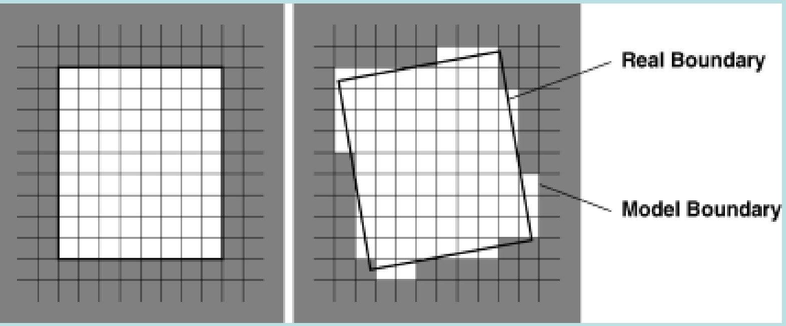
... Unstructured grids have proven to be impractical for climate modelling."

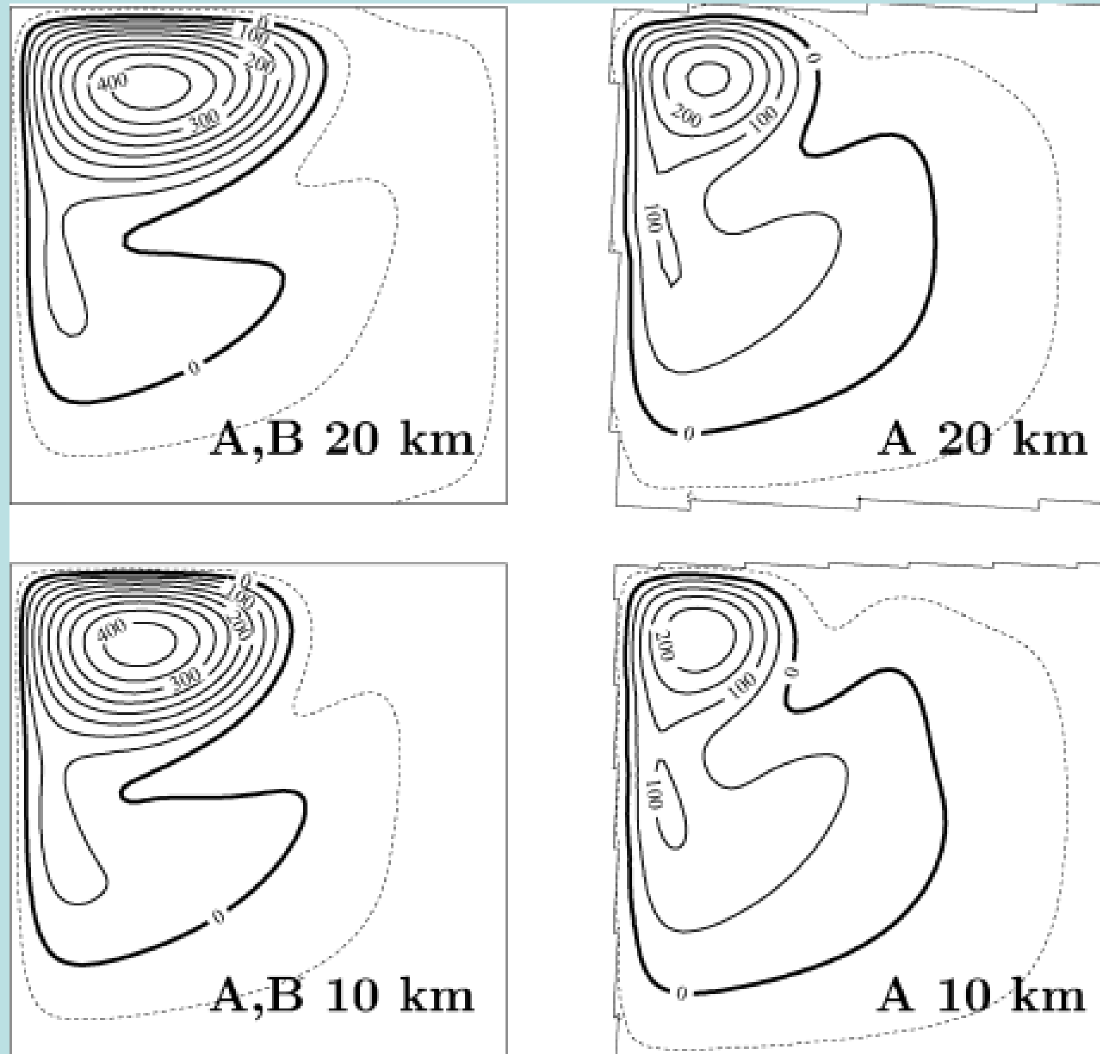
Griffies et al., Development in ocean climate modelling, *Ocean Modelling*, 2000

10-years of FEOM on a grid with mean resolution of 0.5^o



Boundaries are important!



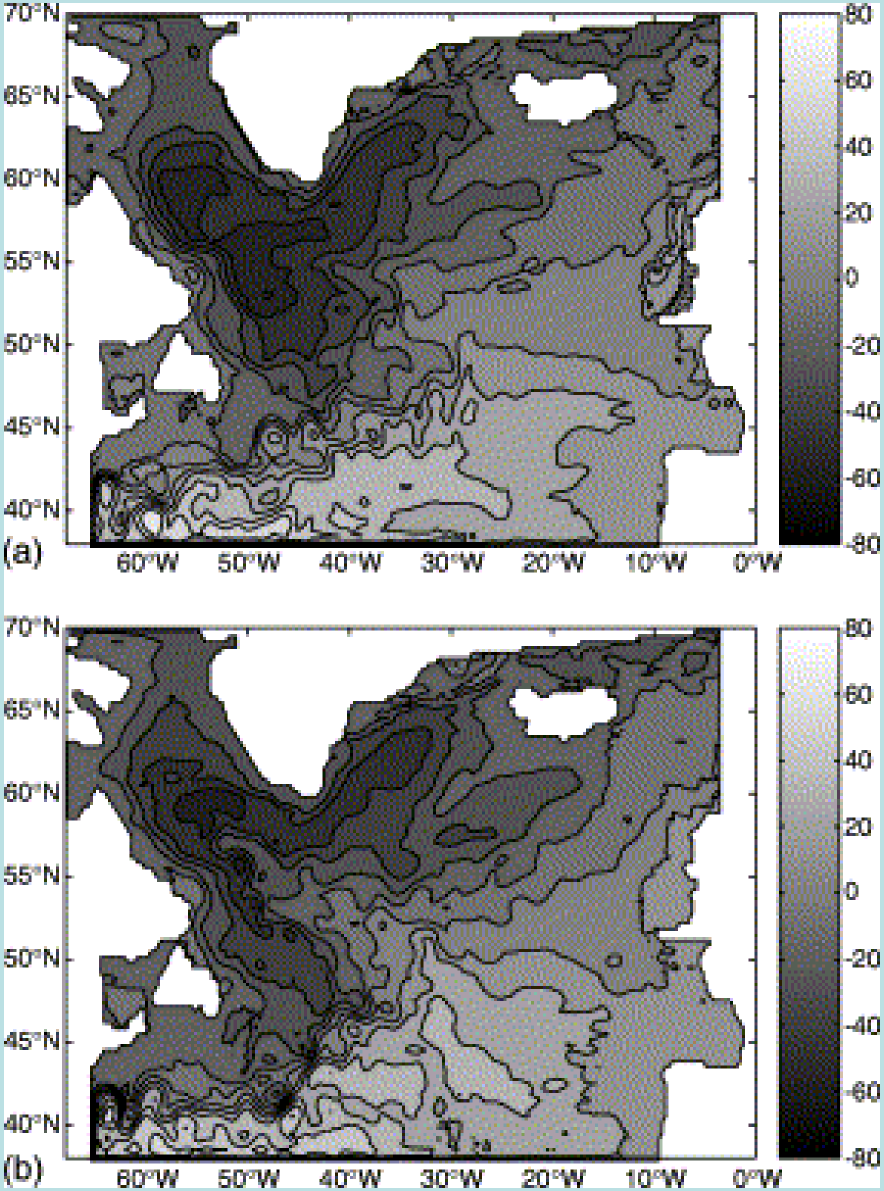


Munk gyre circulation: elevation differs by a factor of two due to stepwise boundaries (angle of rotation is only 3.4 degree)

Dupont et al., 2003

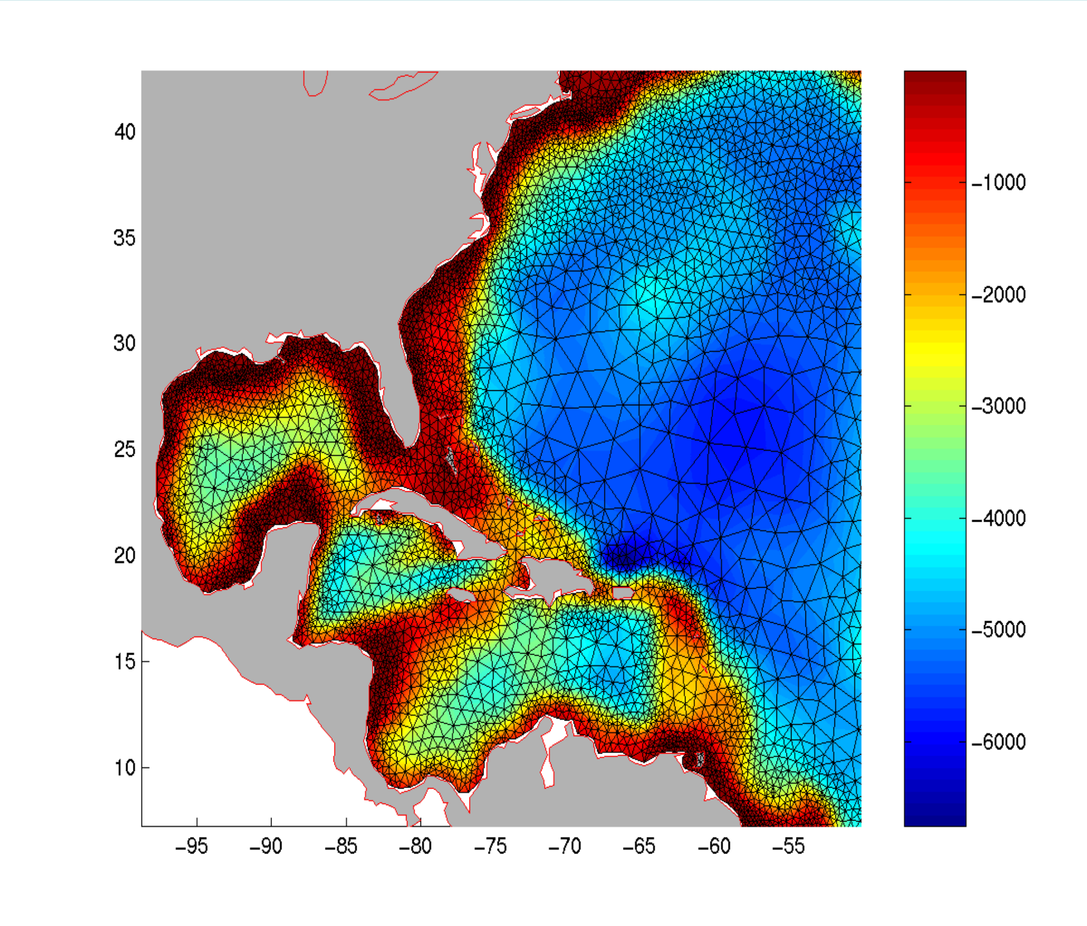
Topography is important!

Mean SSH with MOM full- and partial - cells



Myers & Deacu, 2004

Horizontal discretization with triangles or quadrilaterals:
- accurate representation of coastlines and bathymetry
- flexibility in local mesh refinement (no nesting)
- potential adaptivity (IOM)



Depth, m

Horizontal discretization with elements of different type

--- low order or high order polynomial

--- staggered grids

--- FE or FV

Vertical discretization

like any FD model

z ,sigma, s, hybrid

Perspectives

(a) geometry

In many places the ocean circulation is sensitive to the geometry and bottom topography of the ocean basin (e.g., Denmark Strait, Drake Passage). Unstructured grids seem to provide a tool to explore the role of these features.

(b) global model

Use a coarse global model with local refinement to avoid open boundaries.

(c) adaptivity and error control

'Dynamical' adaptivity seems to be expensive, but 'static' adaptivity is feasible

(d) sea ice modelling

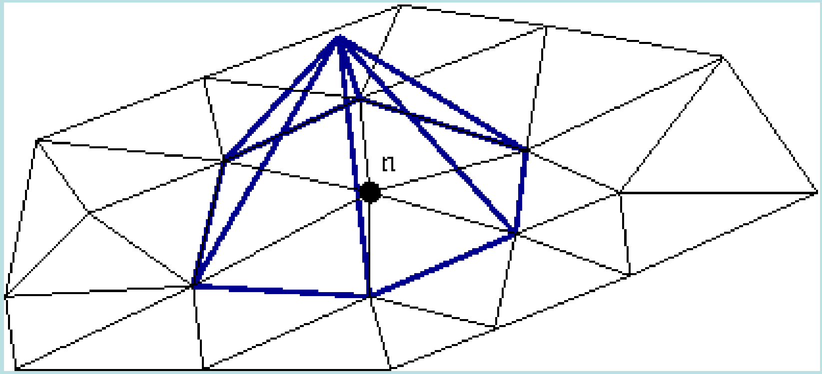
potential for better sea ice rheology, ridging, adaptive refinement

Basics of FE numerics

Representation of variables

2D basis function

Ψ_n



Ansatz for SSH

$$\zeta(x, y) = \sum_{n=1}^{N} \zeta_n \Psi_n(x, y)$$

3D fields are expanded in series of analogous 3D basis functions defined on tetrahedra

Basics of FE numerics

Discretize equations as in spectral methods

- Substitute the expansions for variables in equations
- Require residuals be orthogonal to the basis functions
- Solve for unknown coefficients of the expansions

Variational formulation of the advektion-diffusion equation

$$\partial_{\mathrm{t}} \mathrm{T} + \mathbf{v} \cdot \operatorname{grad} \mathrm{T} + \operatorname{div} \mathbf{K} \operatorname{grad} \mathrm{T} = 0$$

project the equation onto piecewise polynomial functions, multiply by testfunction $\tilde{T}$ and integrate

$$\int_{\Omega} \partial_t T \tilde{T} + \mathbf{v} \cdot \operatorname{grad} T \tilde{T} - \operatorname{div} \mathbf{K} \operatorname{grad} T \tilde{T} \, \mathrm{d}\Omega = 0$$

Partial integration and Gauß-Theorem

$$\int_{\Omega} \partial_t T \, \tilde{T} \, \mathrm{d}\Omega \, +$$

Massmatrix
(symmetric)

Stiffnessmatrix
(unsymmetric
by **v**-entries)

$$+ \int_{\Omega} \mathbf{v} \cdot \operatorname{grad} T \, \tilde{T} + \operatorname{grad} \tilde{T} \, \mathbf{K} \operatorname{grad} T \;\; \mathrm{d}\Omega \, -$$

$$- \int_{\partial\Omega} \tilde{T} \, \mathbf{K} \operatorname{grad} T \cdot \mathbf{n} \, \mathrm{d}\Gamma = 0$$

boundary conditions
(this integral is evaluated
and put to the
right hand side)

Basics of FE numerics

1D example:

- Equation

$$\partial_t T + u \partial_x T = 0$$

- FE-discretization on a uniform grid with grid size h

$$\partial_t \left(\frac{T_{n-1} + 4T_n + T_{n+1}}{6} \right) + u \left(\frac{T_{n+1} - T_{n-1}}{2h} \right) = 0$$

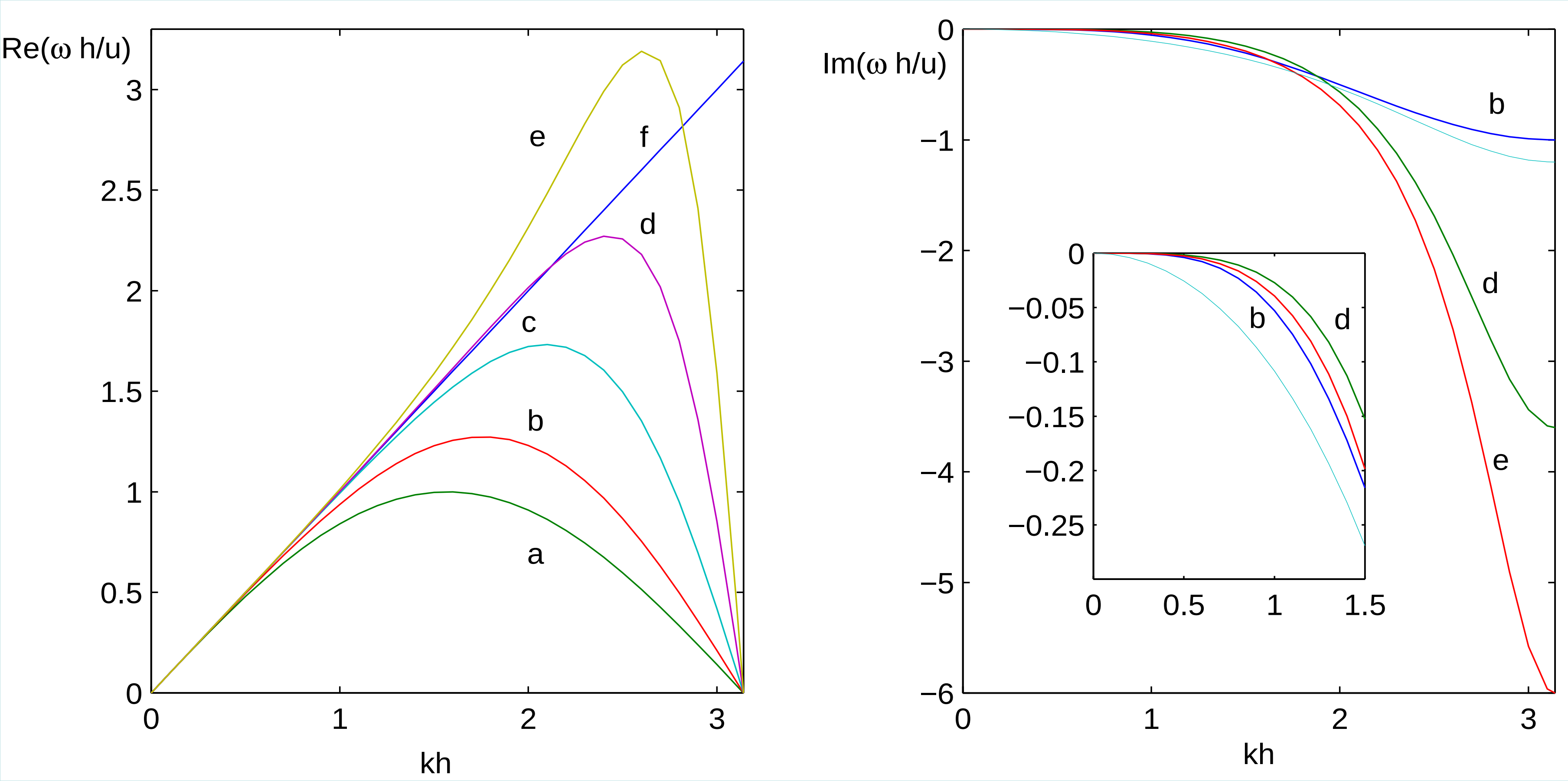
Weighting over neighbours — Central differences

☺ same stencil for all terms ⬇ reduced dispersion

☹ necessity of matrix inversion

Basics of FE numerics

Advection scheme



(a) – central differences, (b) QUICK, (c) – unstabilized FE, (d) – stabilized FE, (e) – overstabilized FE; (f) – the exact dispersion

Finite elements vs. finite differences:

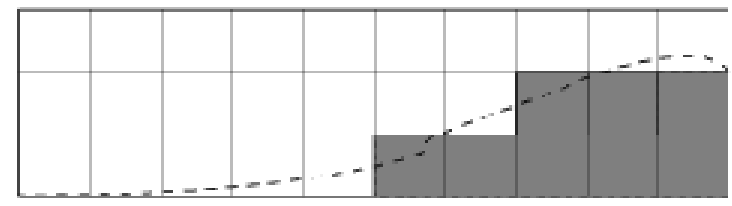
Need for assembling matrices

Need for effective preconditioners and solvers

On unstructured grids computing RHS is more expensive than with finite difference method

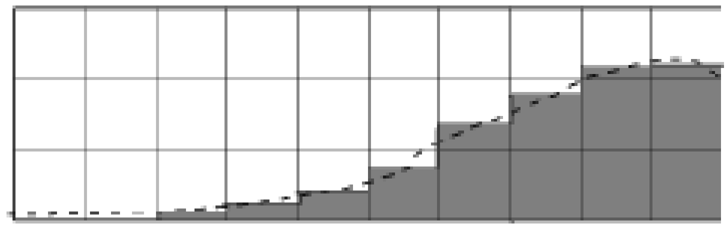
Parallelization of matrix assembly and RHS computation is relatively straightforward, effective parallelization of factorization and solvers is feasible, but requires special efforts.

Vertical discretization

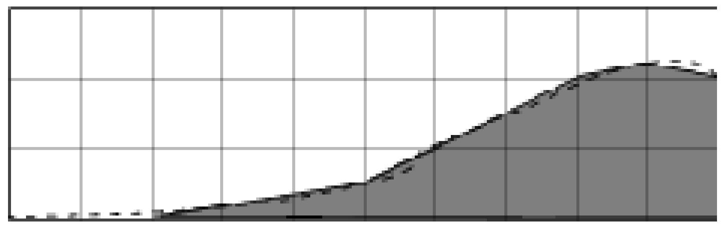


Full cells (generally used, no pressure gradient errors)

At the cost of pressure gradient errors in the lowest cells:



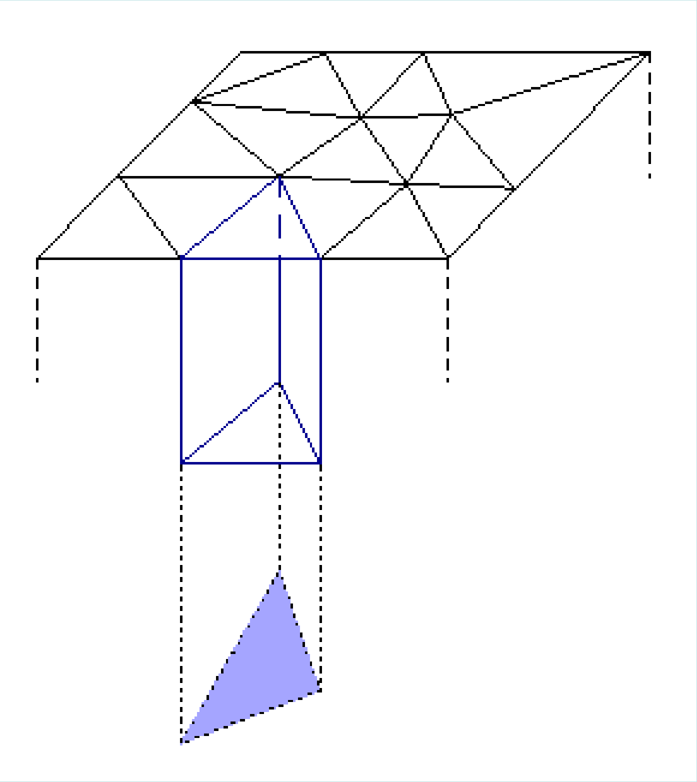
Partial cells (a few examples)



Shaved cells (not yet used in climate studies)

Vertical discretization

Surface triangle defines
a prism



Possibilities to proceed:

(a) full prisms and z-levels (analogous to MOM, POP, HOPE, MITgcm, OPA)

⇓ stepwise bottom

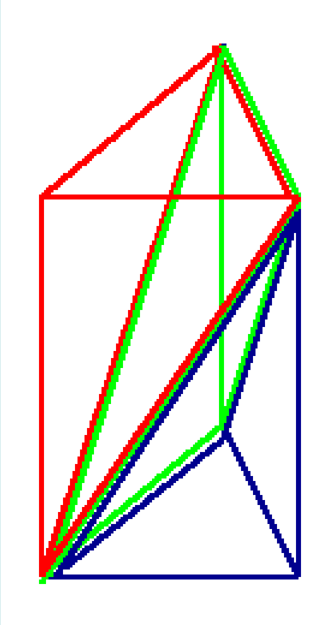
(b) full prisms and terrain following levels (analogous to POM, ROMS)

⇓ pressure-gradient errors

(c) cut bottom prisms, and z-levels (analogous to shaved cells of MITgcm)

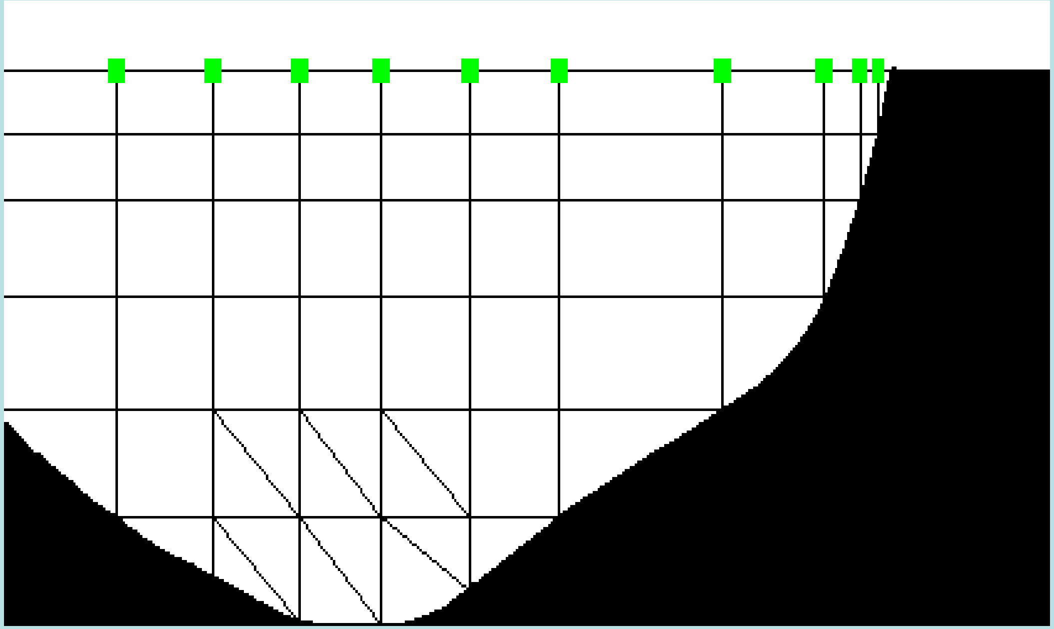
Vertical discretization

A full prism is divided into three tetrahedra



A prism cut by bottom is represented by one or two tetrahedra

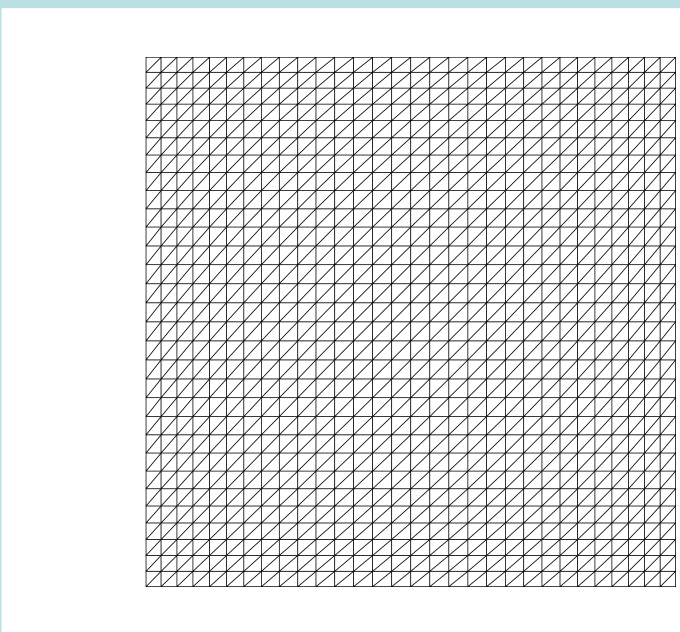
No errors in pressure gradients due to variable horizontal resolution

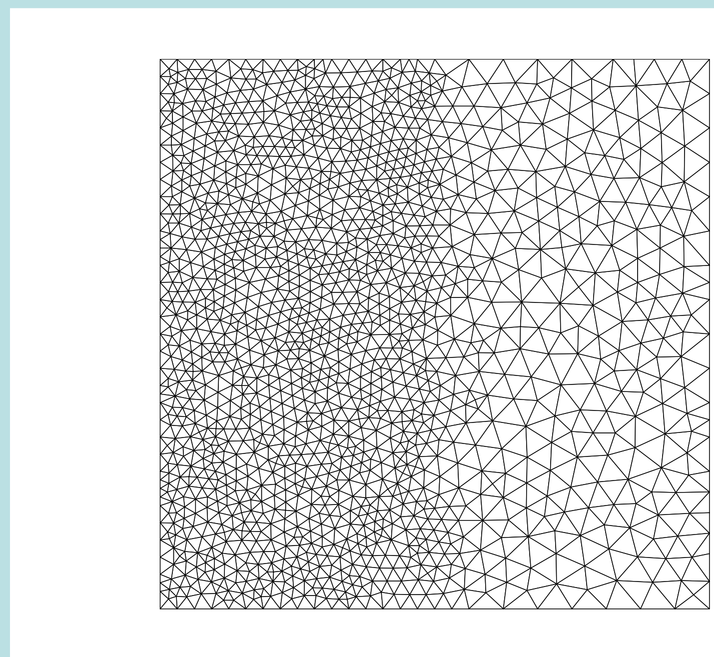


FEOM uses tetrahedra - the most flexible yet expensive way

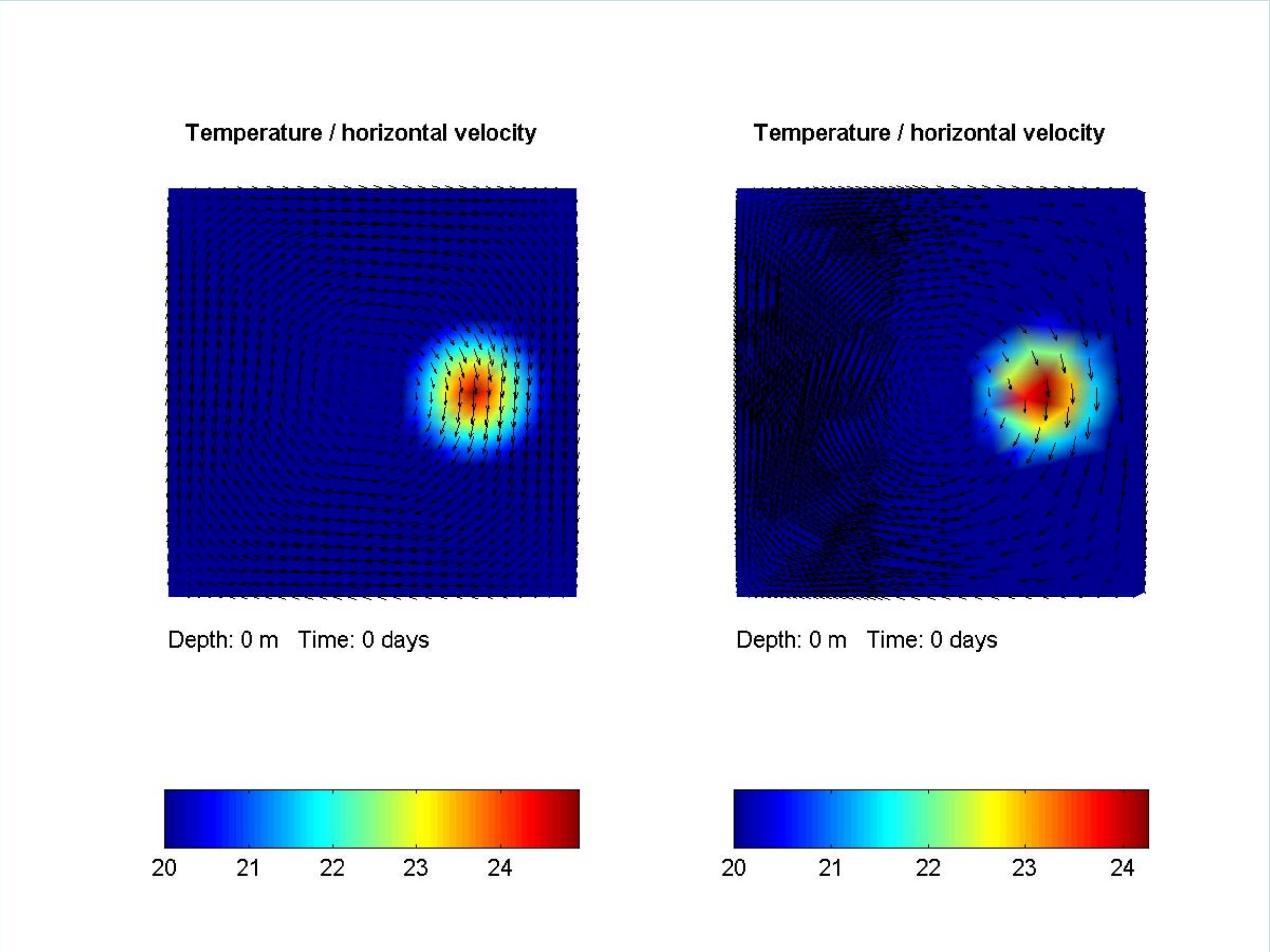
Advection of a temperature anomaly

Gaußian temperature anomaly in a divergence free velocity field in a regular and unstructured mesh

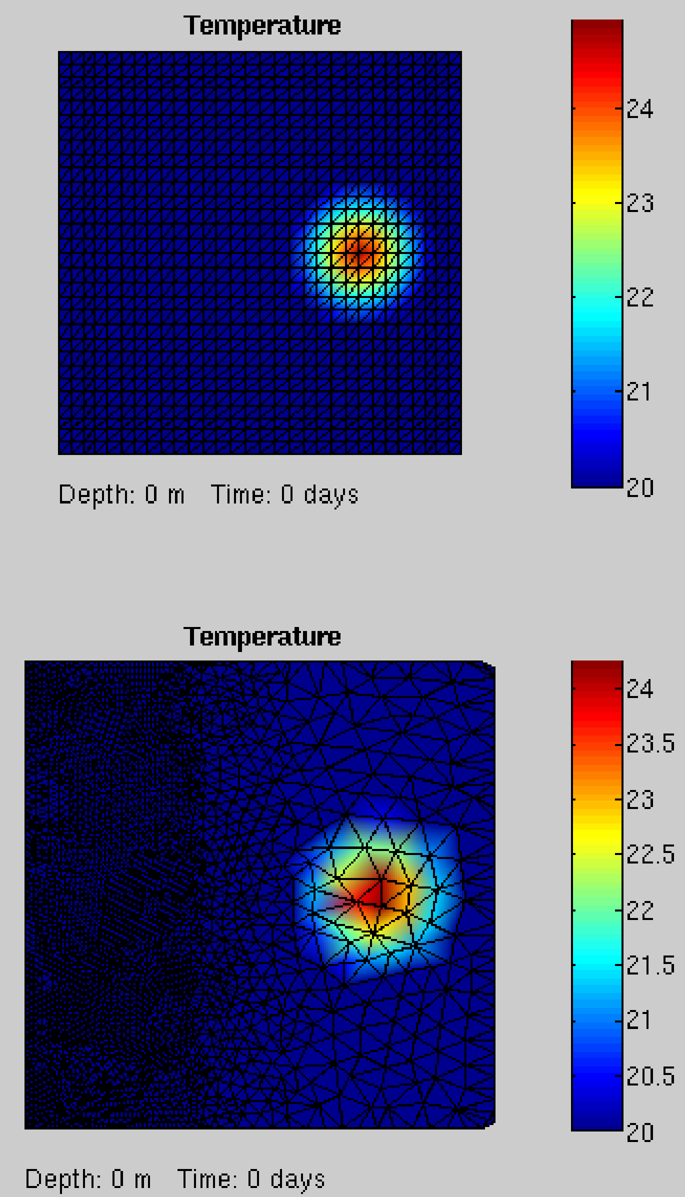




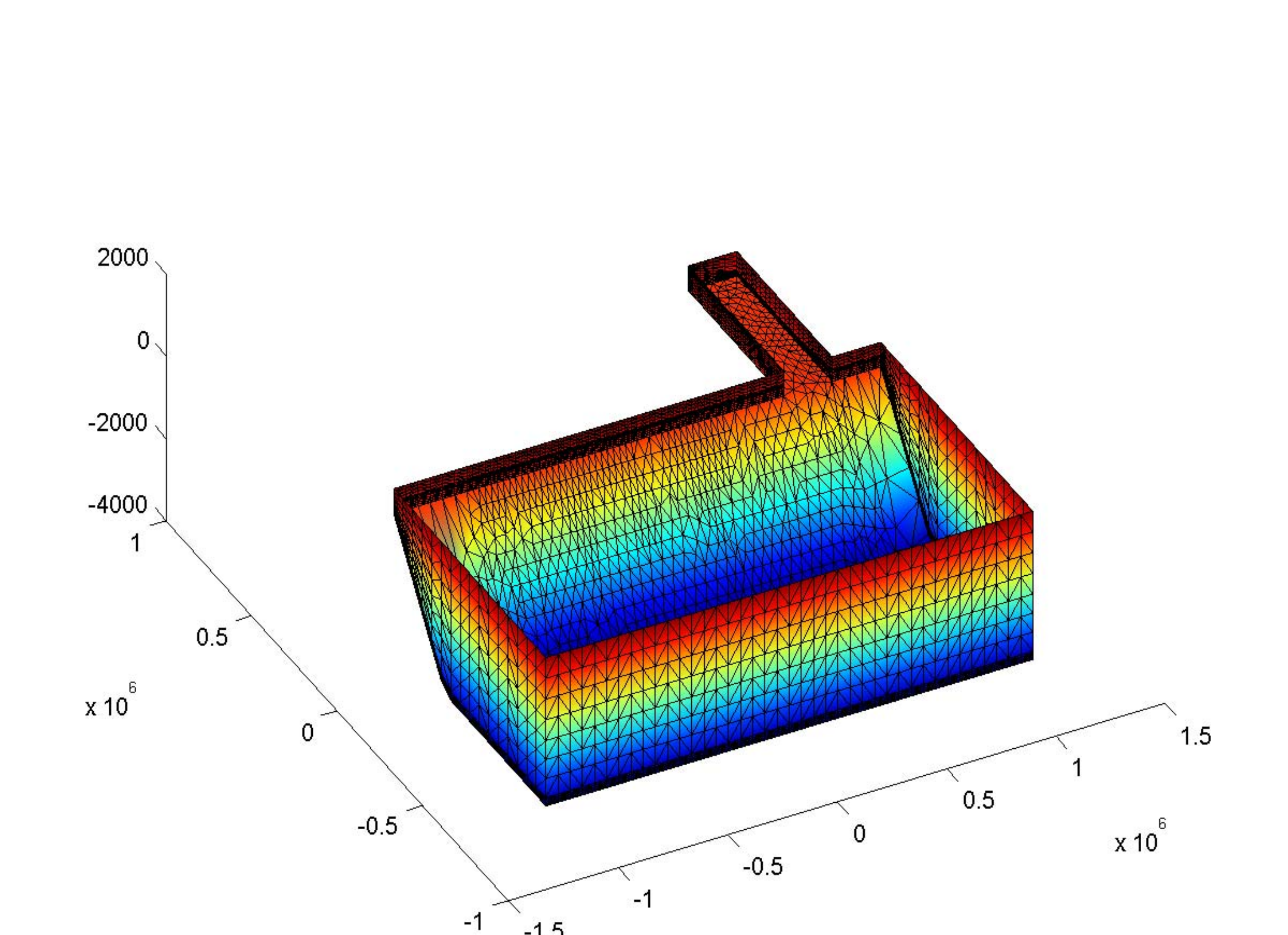
velocity field



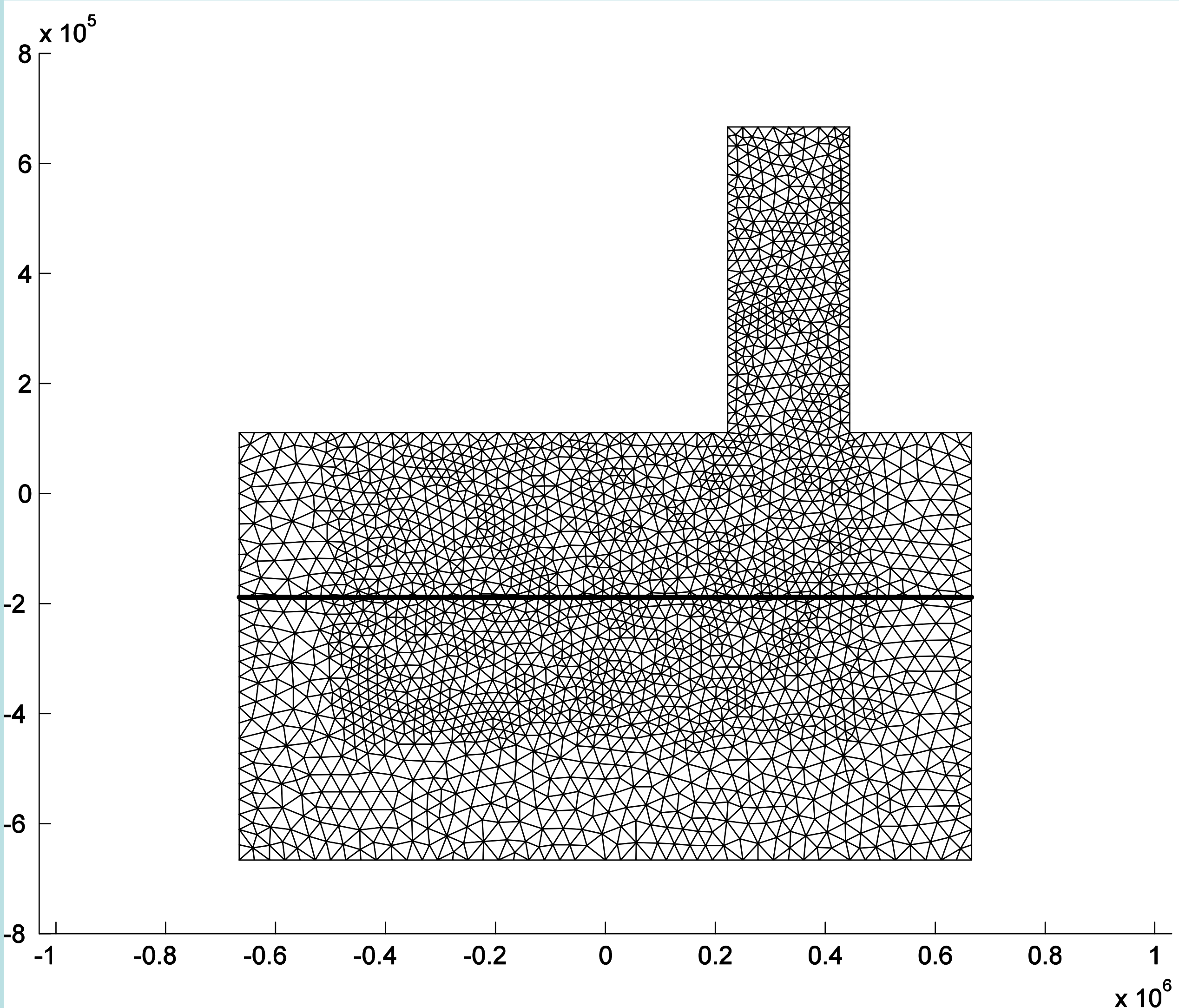
3D, but independent of depth


Temperature
24
23
22
21
20
Depth: 0 m Time: 0 days
Temperature
24
23.5
23
22.5
22
21.5
21
20.5
20
Depth: 0 m Time: 0 days

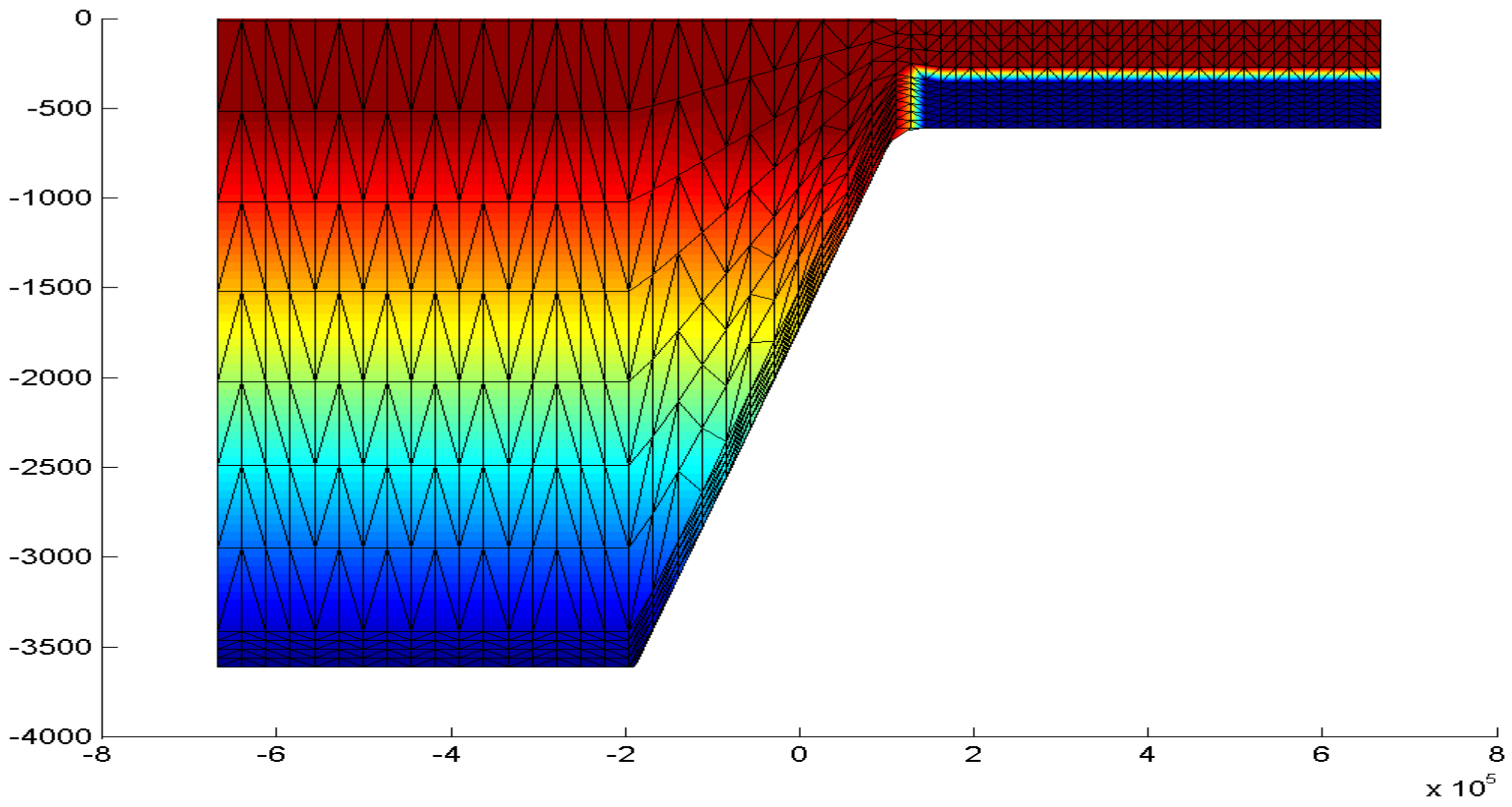
application to the DOME setup (Dynamics of Overflow Mixing and Entrainment)



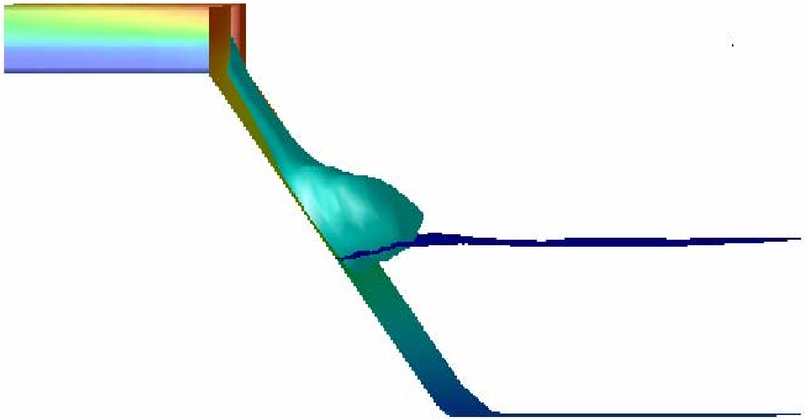
Surface grid



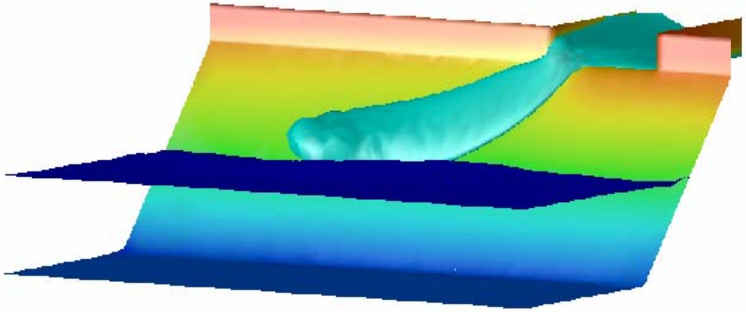
Vertical discretization and initial temperature stratification



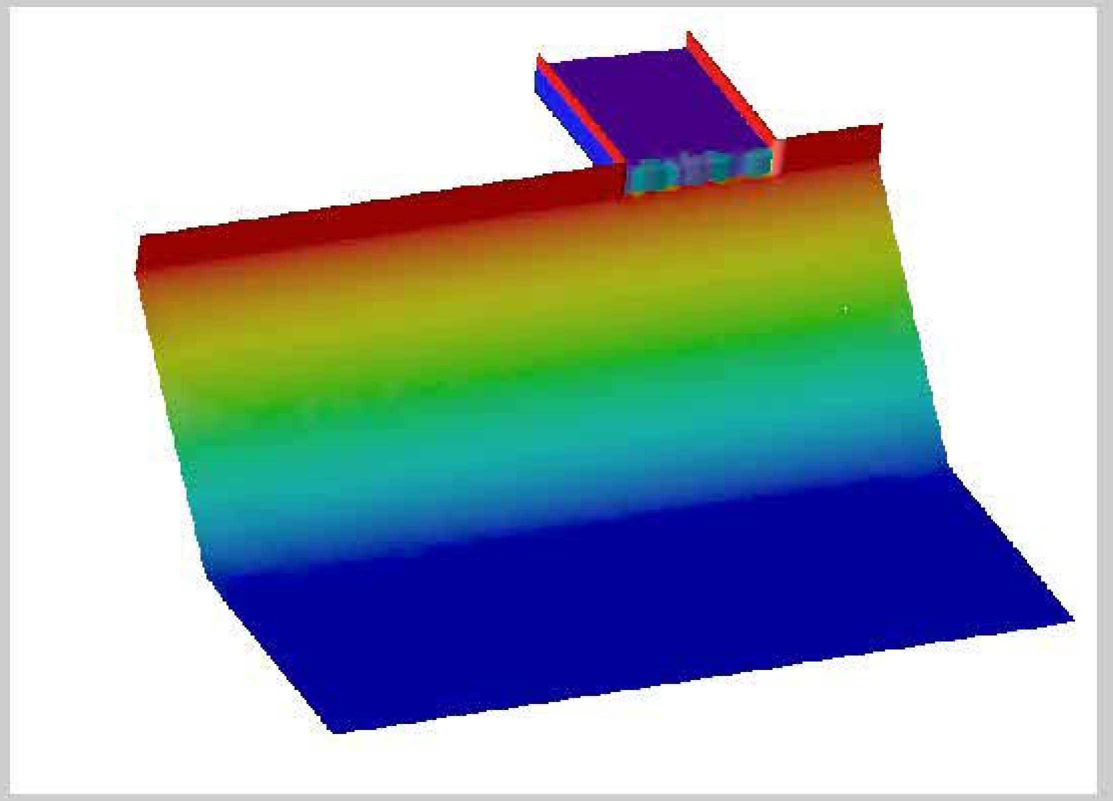
Isosurface of marker

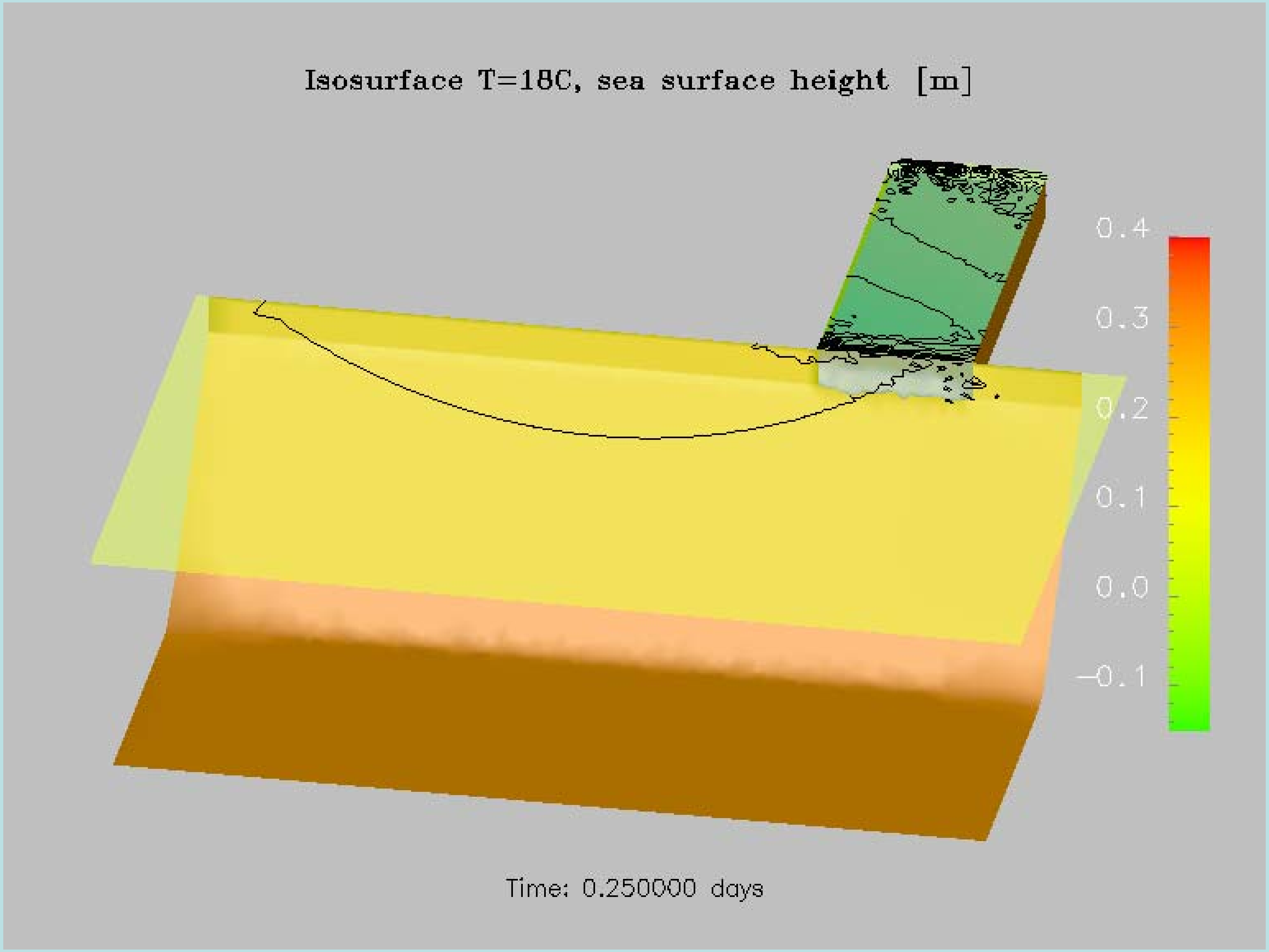


overflow density difference: 2 kg/m3 – Tracer isosurface: 25 % – Temperature isosurface: 14.9 deg C



overflow density difference: 2 kg/m3 – Tracer isosurface: 25 % – Temperature isosurface: 14.9 deg C – slope: 1 %

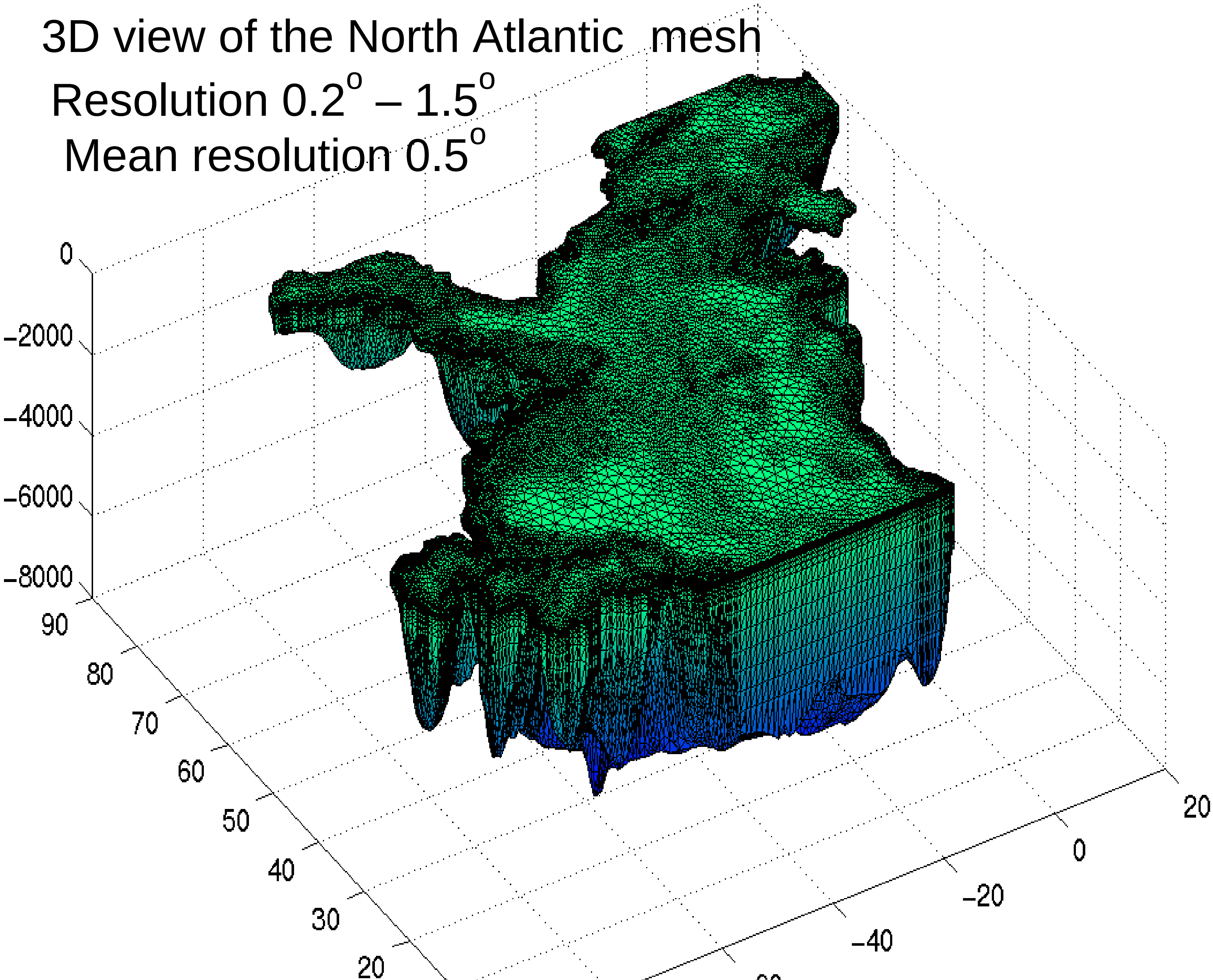


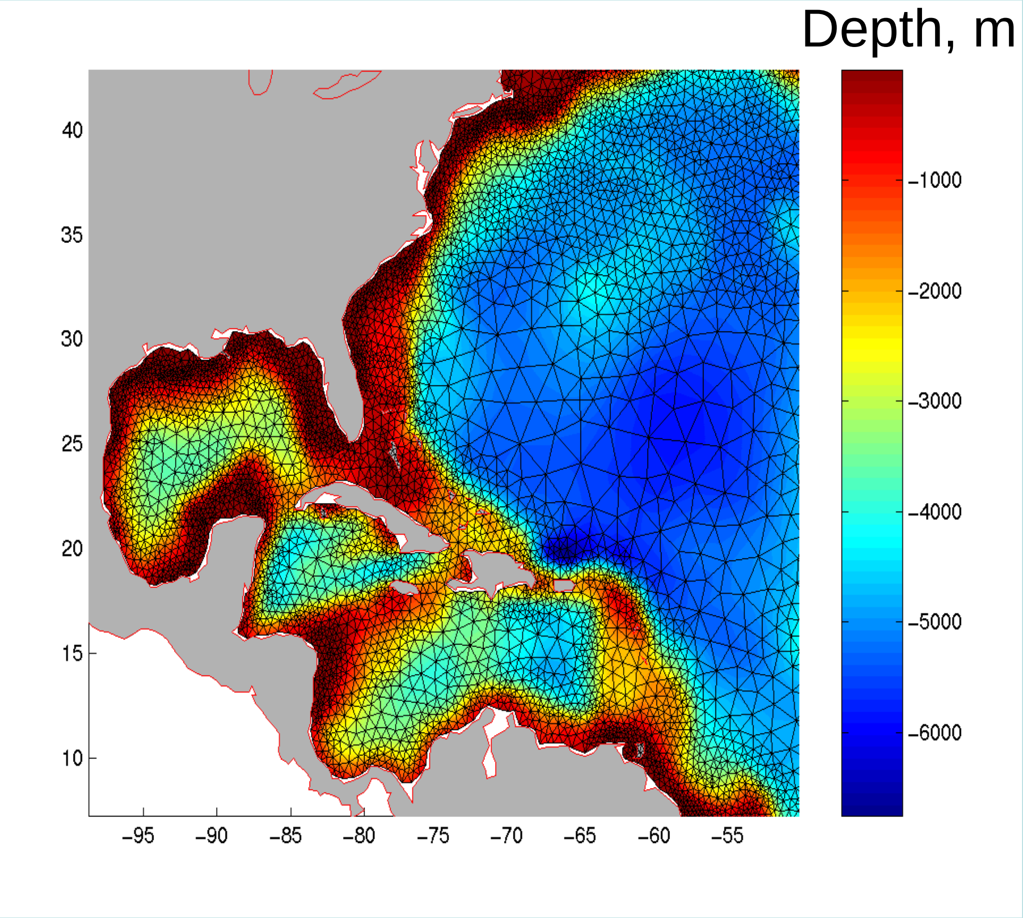

Isosurface T=18C, sea surface height [m]
0.4
0.3
0.2
0.1
0.0
−0.1
Time: 0.250000 days

3D view of the North Atlantic mesh

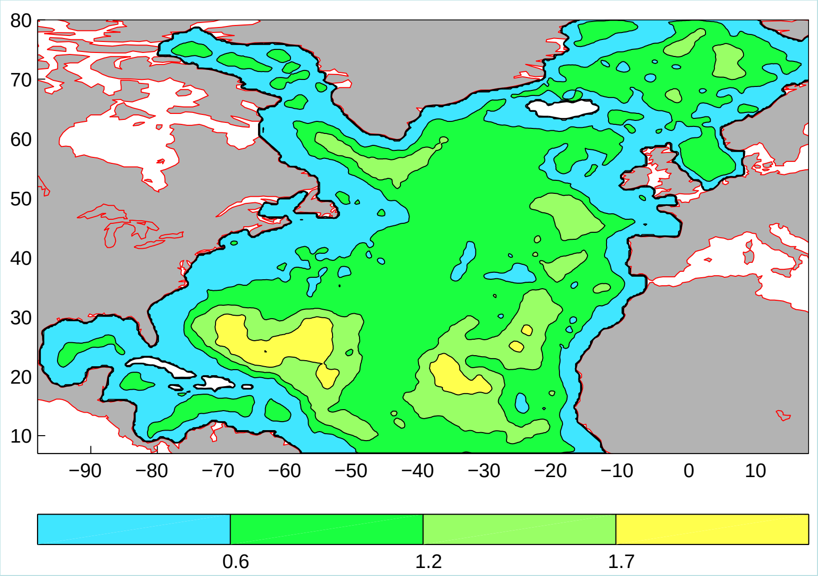
Resolution 0.2^{o} – 1.5^{o}

Mean resolution 0.5^{o}





Resolution in degs



Horizontal discretization with triangles:

--- accurate representation of coastal lines

--- flexibility in local mesh refinement

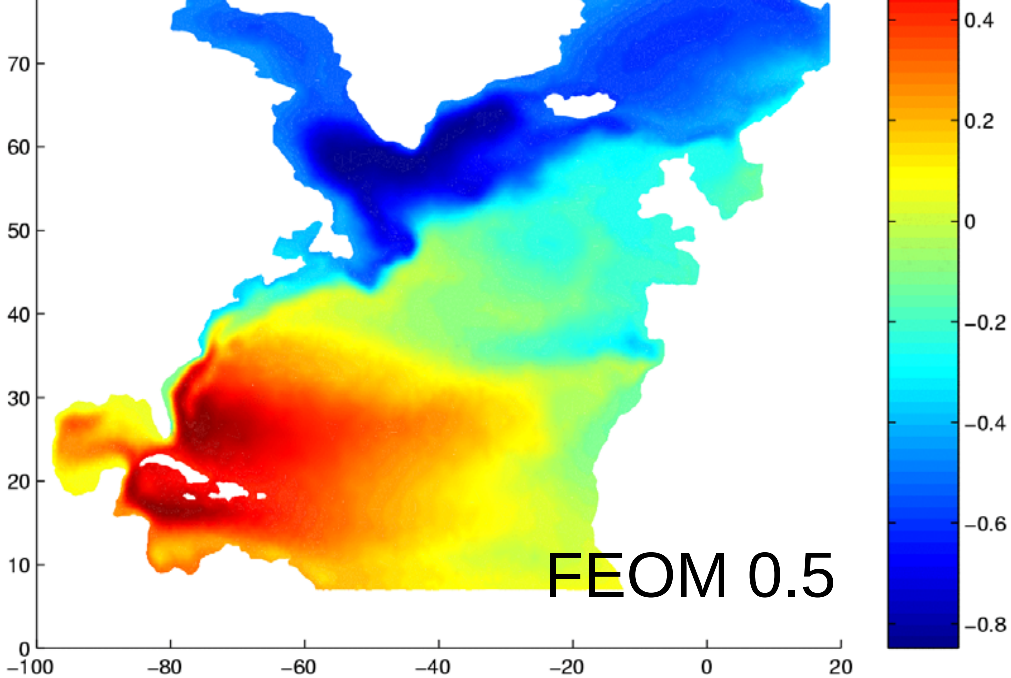
--- potentiality in adaptivity (not yet used)

Basic features of FEOM

- Primitive equations
- Rigid lid and free surface options
- 2D unstructured triangular mesh
- Vertically aligned nodes
- Tetrahedral elements
- z-levels with inclined bottom (any level system can be used in principle without modifying the code)
- Backward Euler time stepping (to be replaced with Cranck-Nicolson method)

Specific features of the NA version

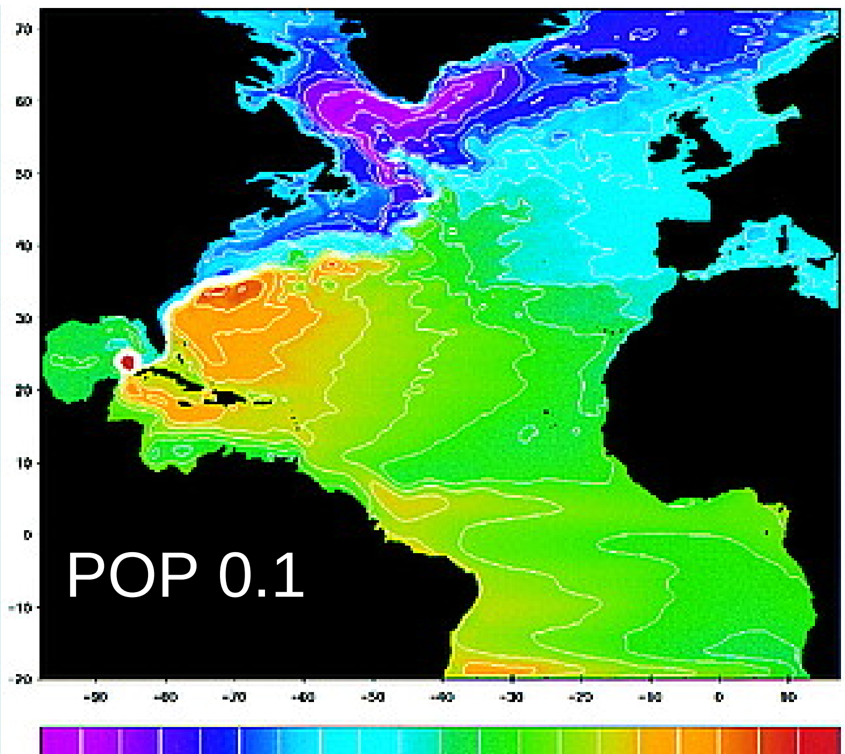
- Richardson number dependent vertical diffusion
- Convection via enhanced diffusion (1 m^2/s)
- Smagorinsky horizontal viscosity
- Background horizontal viscosity and diffusion 25 m^2/s
- 0.2 ° - 1.5 ° resolution (16000 surface nodes)
- 23 z-levels (220000 3D nodes)
- Time step 2 h for (u, v, ζ) and 1 h for (T, S) in the rigid lid mode

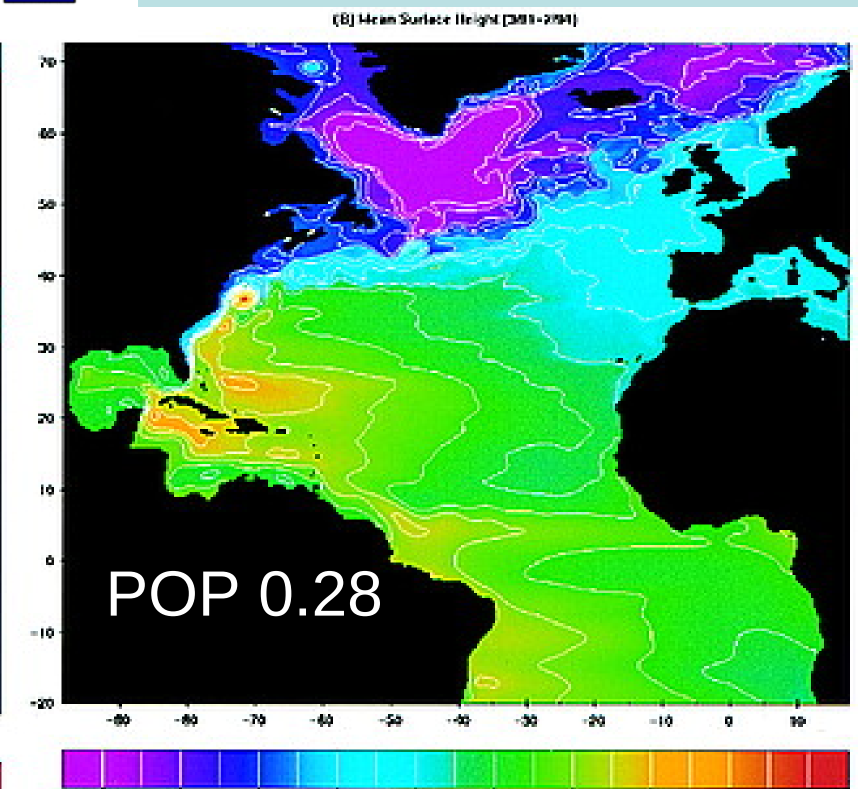


Mean SSH

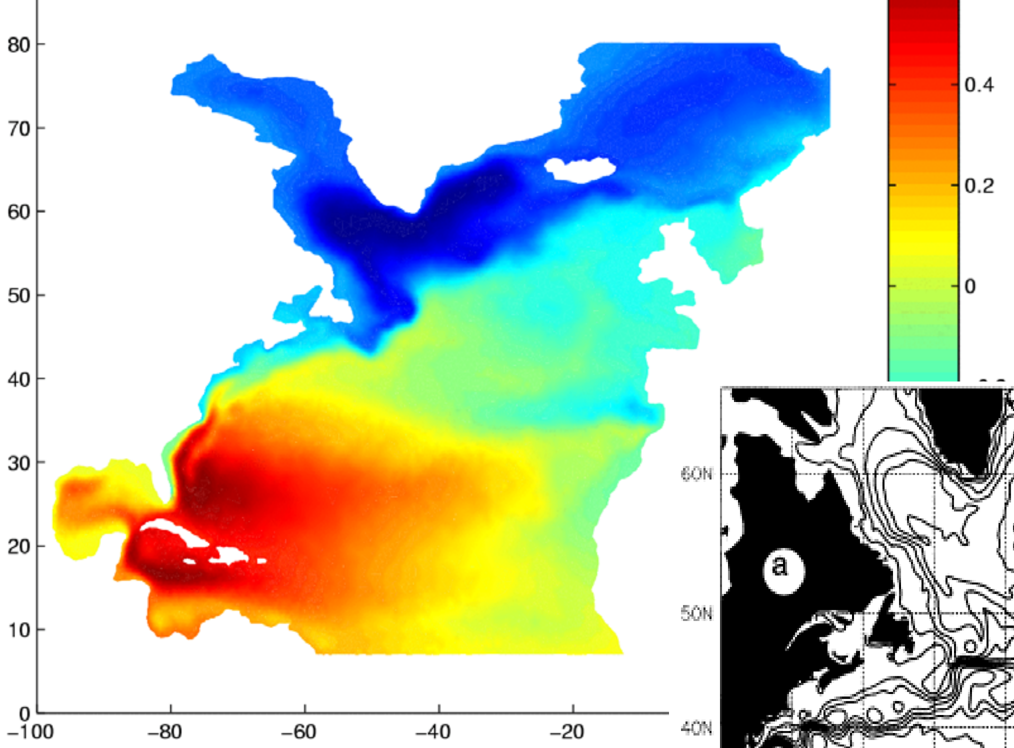
POP 0.1^{o} >FEOM 0.5^{o}>POP 0.28^{o}

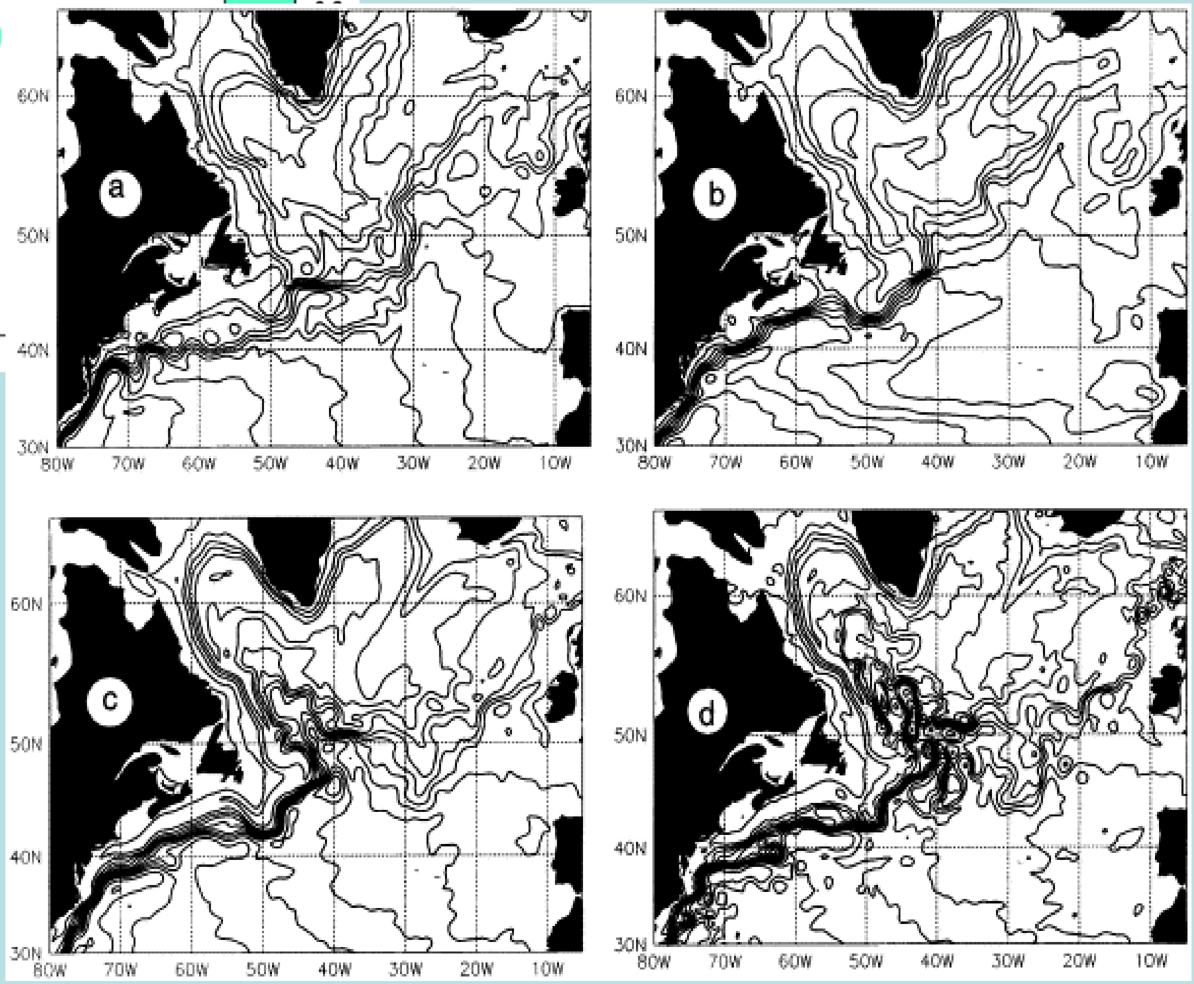
- •Azores Current
- •Recirculation
- •NA current
- •Sub-polar gyre





Comparison with DYNAMO models



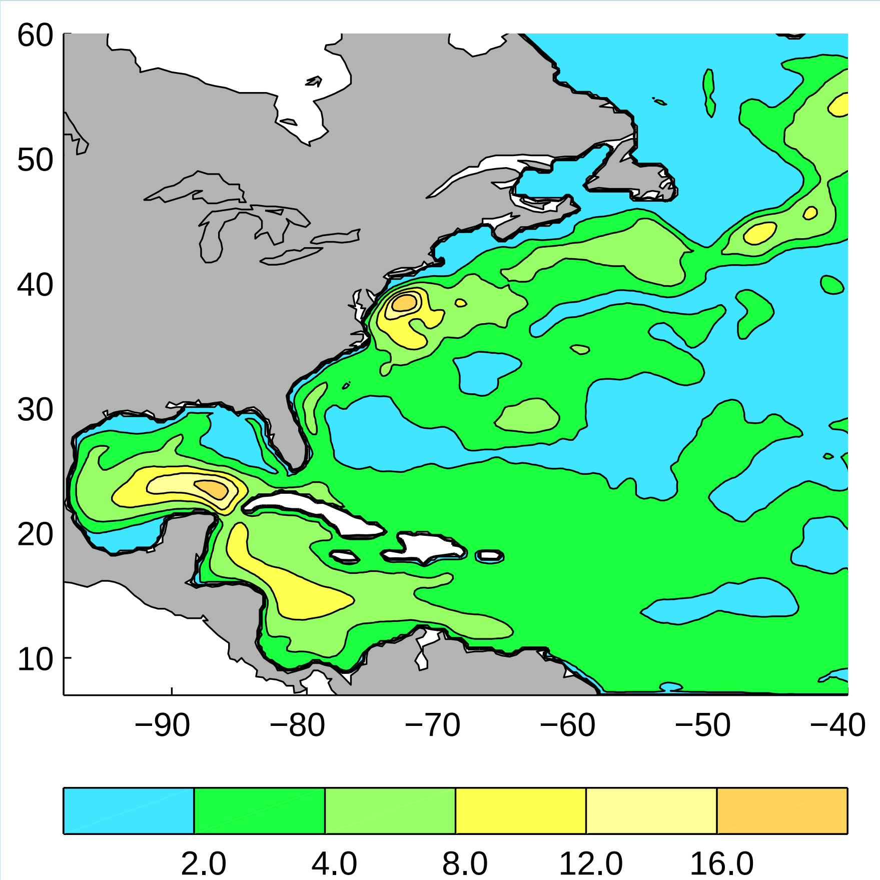


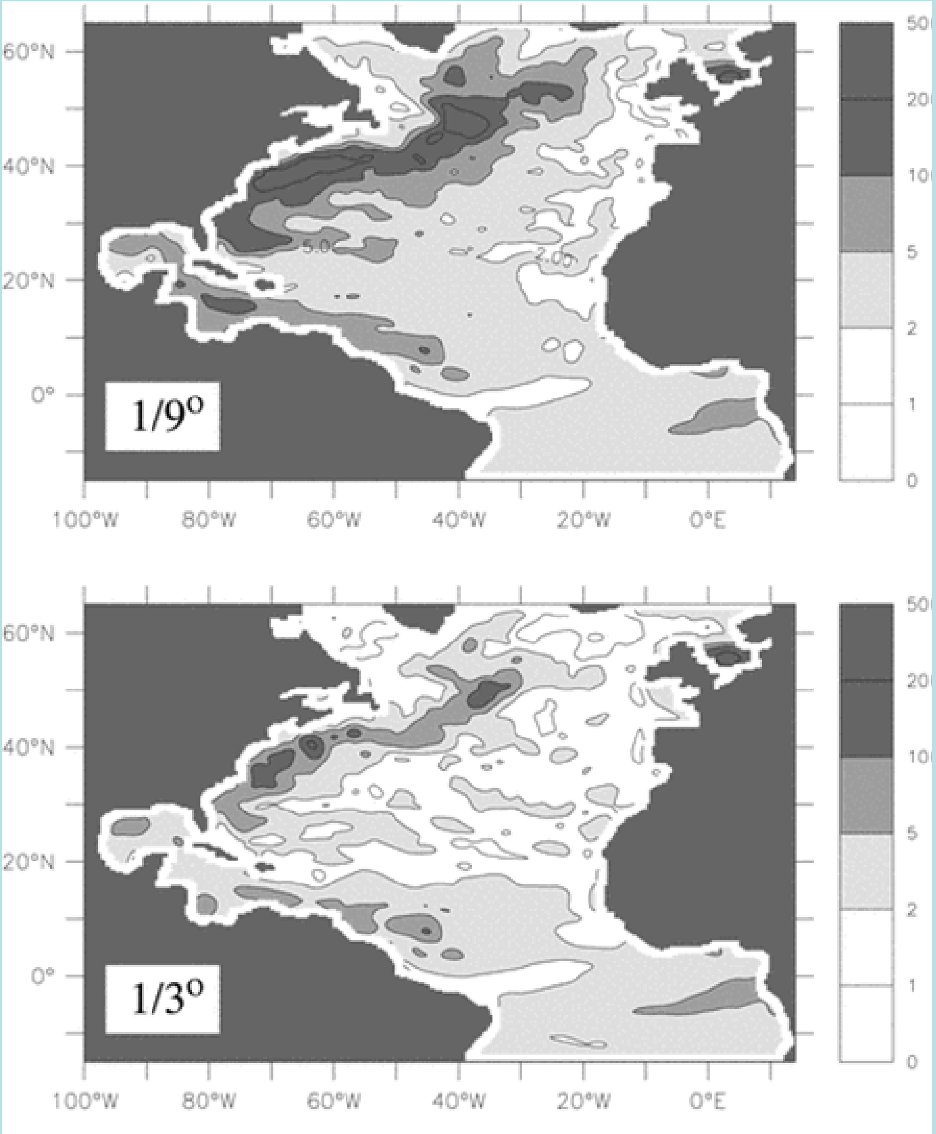
(a) LEVEL
(b) ISOPYCNIC
(c) SIGMA
(d) SIGMA (snapshot)

SSH variability

MOM (Oschlies, 2002)

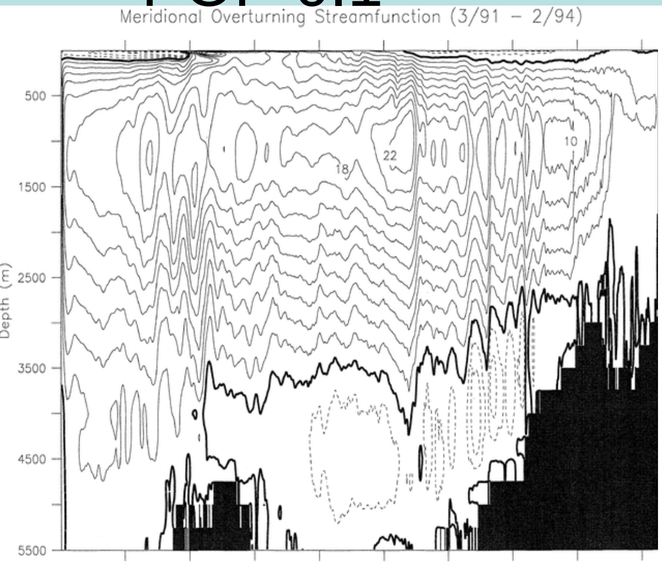
FEOM



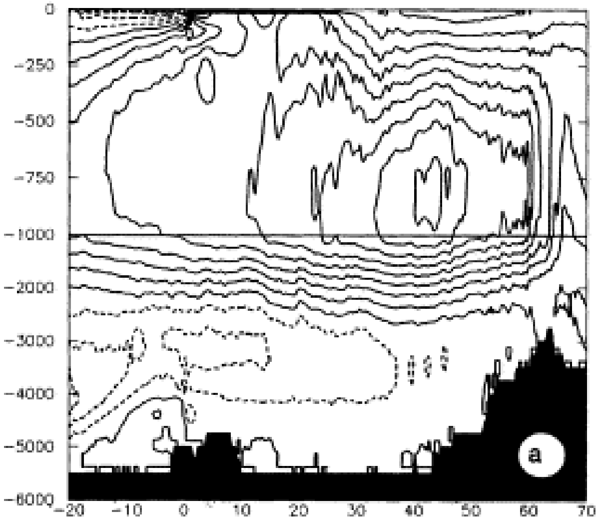


Meridional overturning

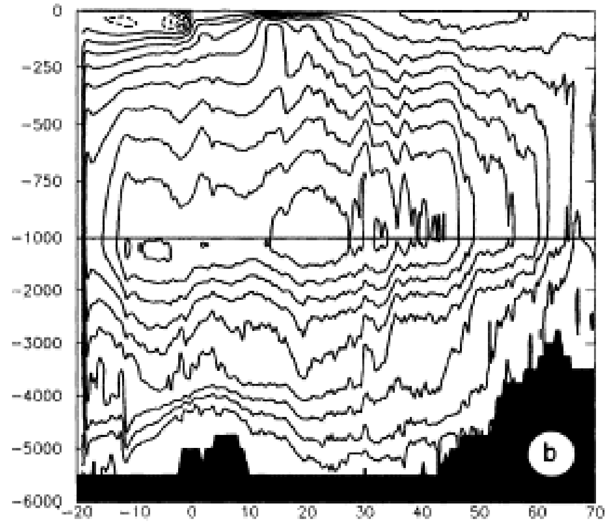
POP 0.1°

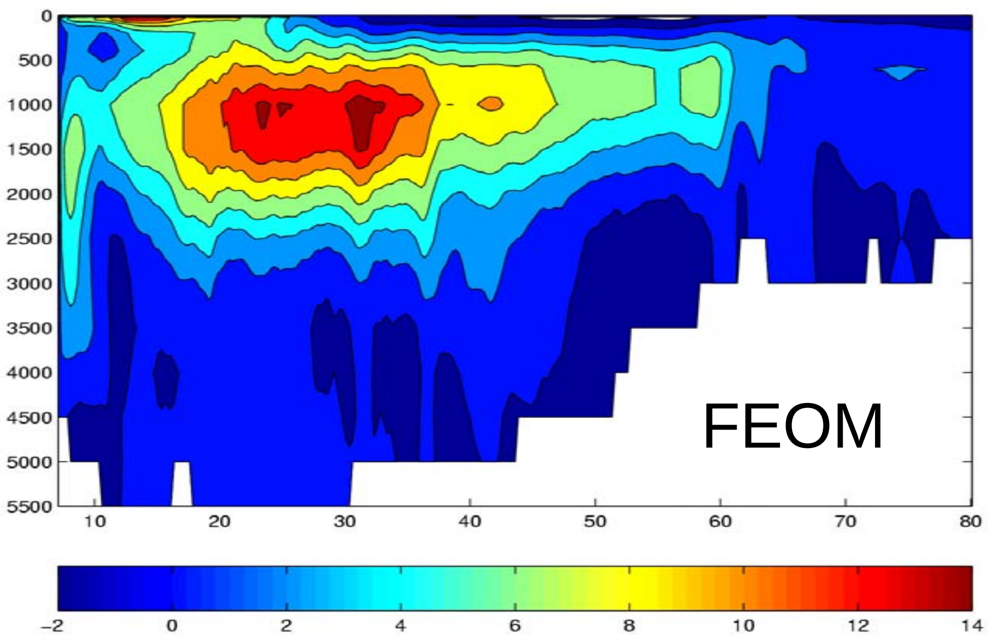


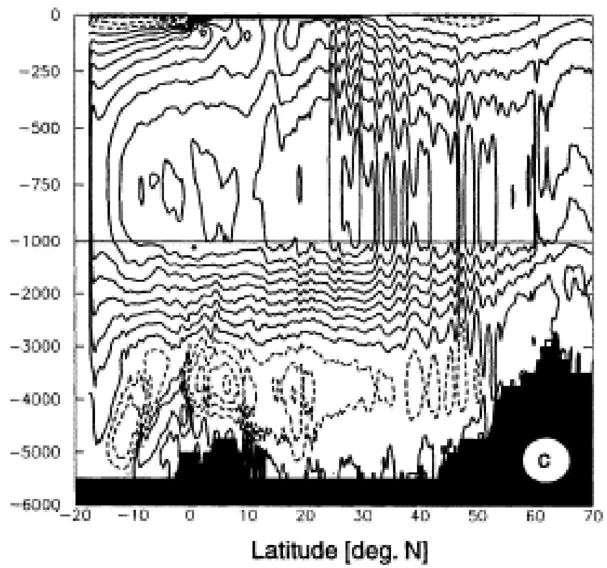
DYNAMO MOM



DYNAMO MICOM

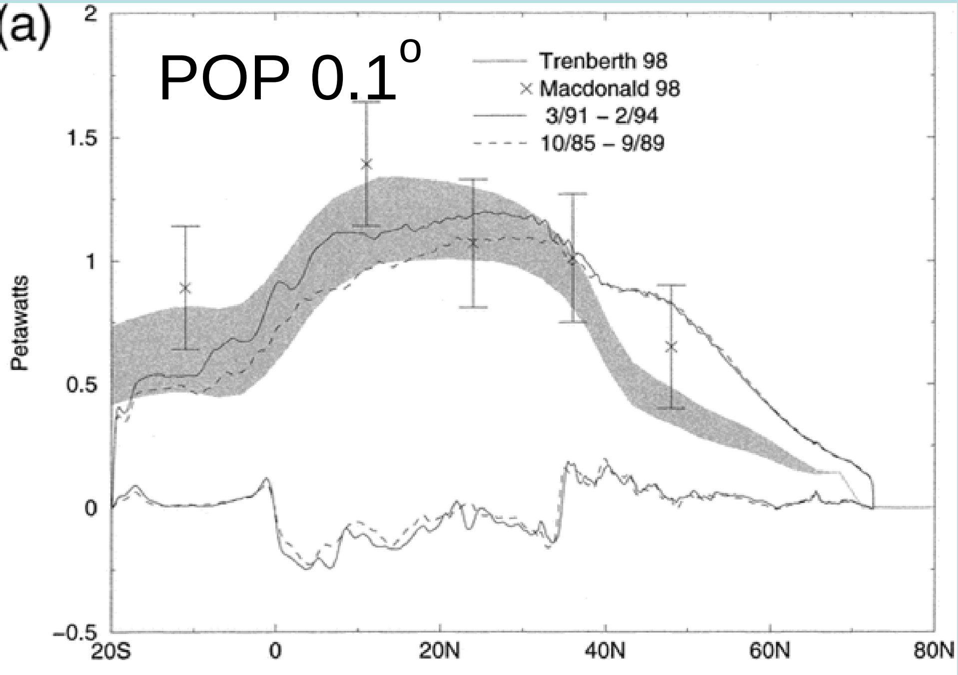


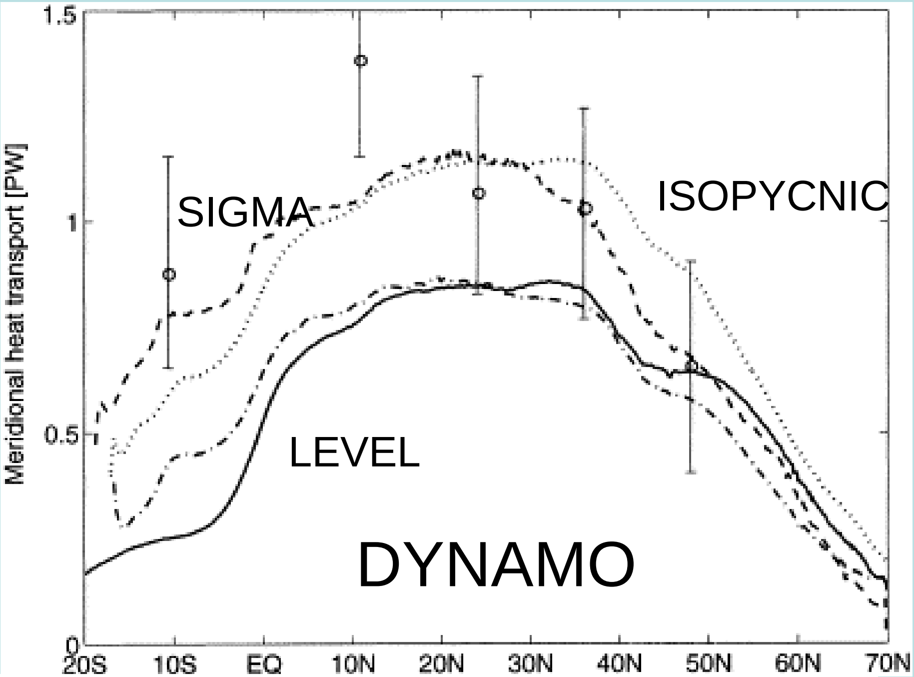




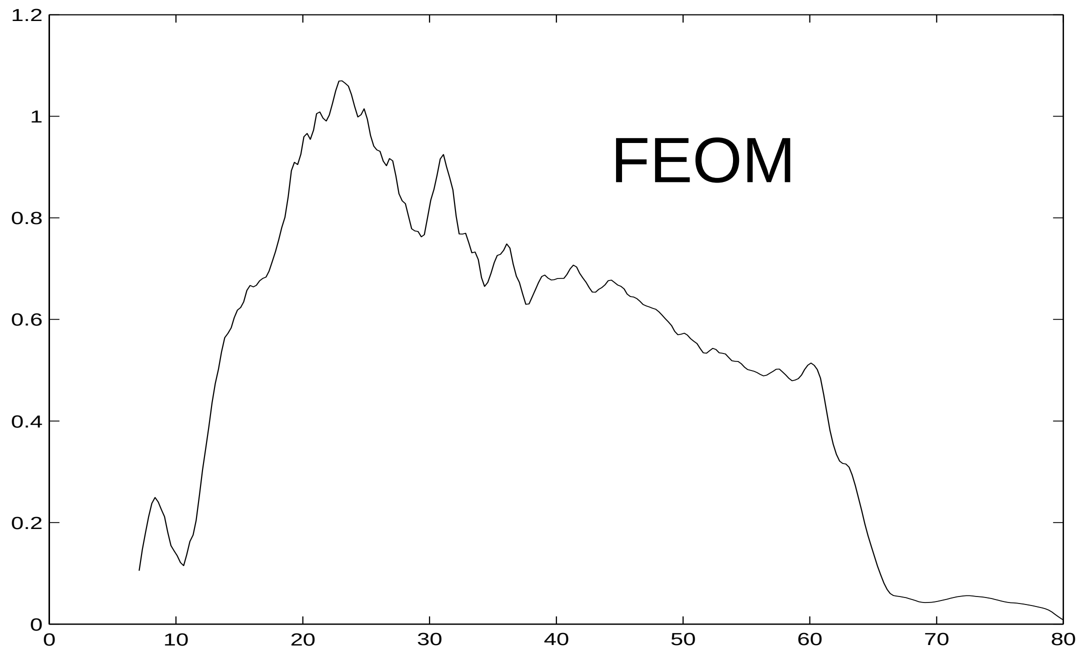
DYNAMO SPEM

Meridional heat transport

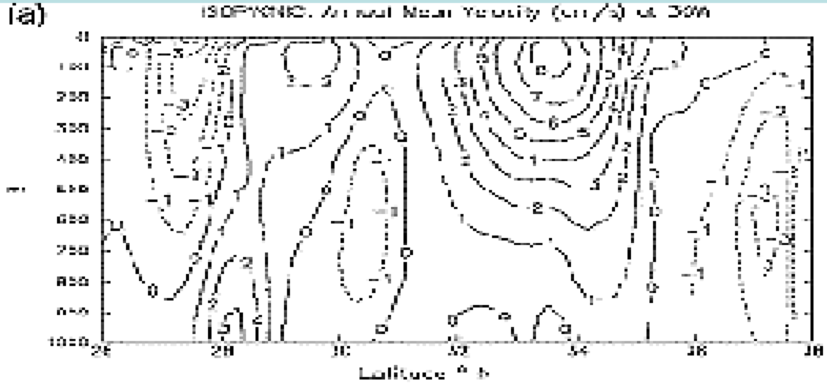




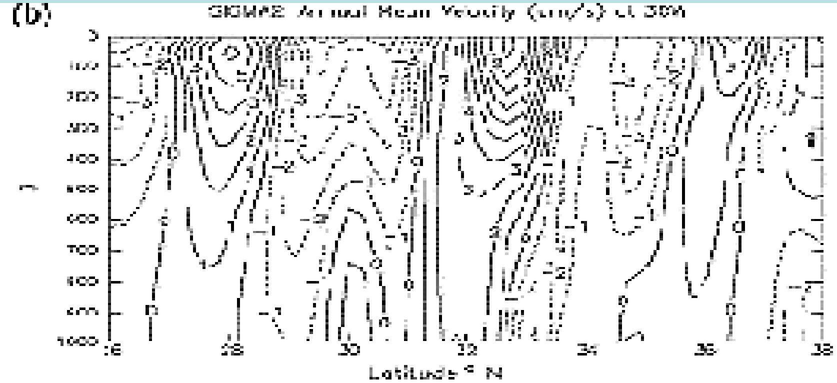
Low heat transport south of 20 N is due to closed open boundaries



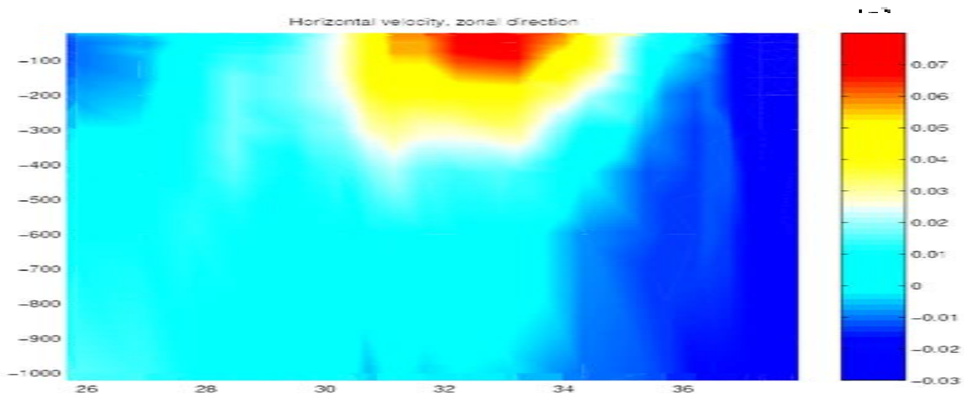
Zonal velocity cross-section at 30^{o} W (Azores Current)



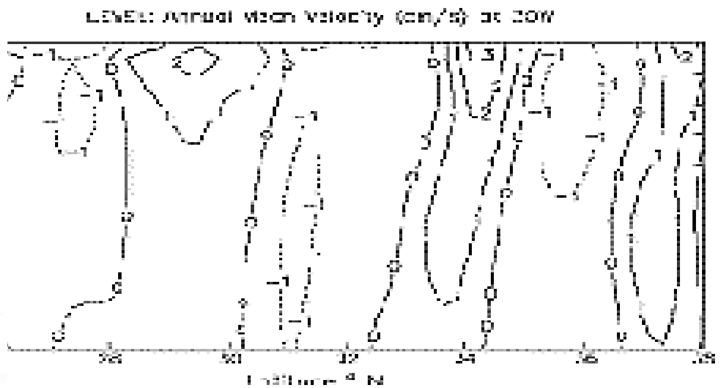
ISOPYCNIC



SIGMA

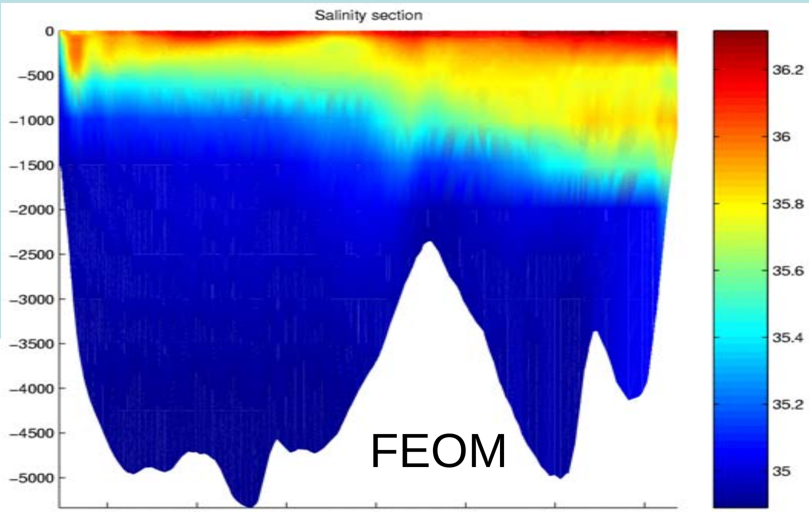


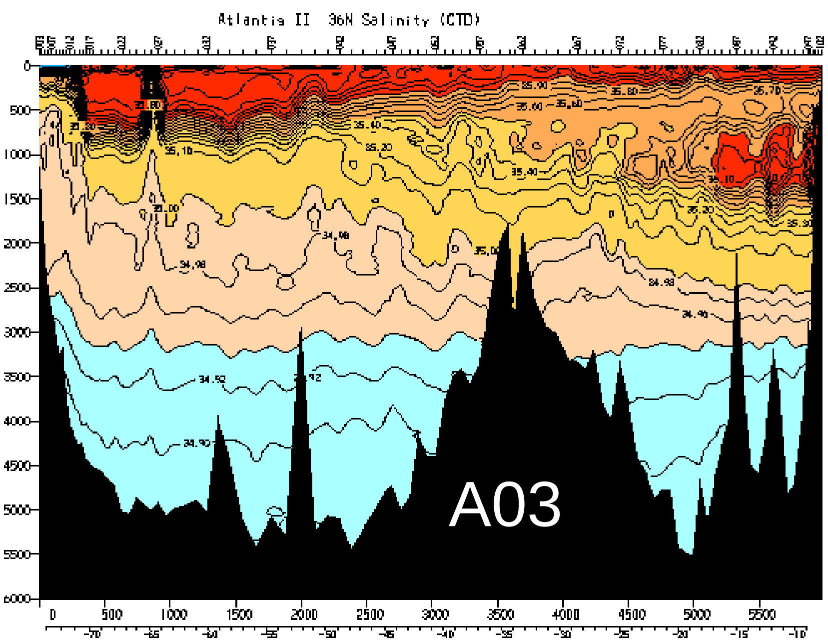
FEOM

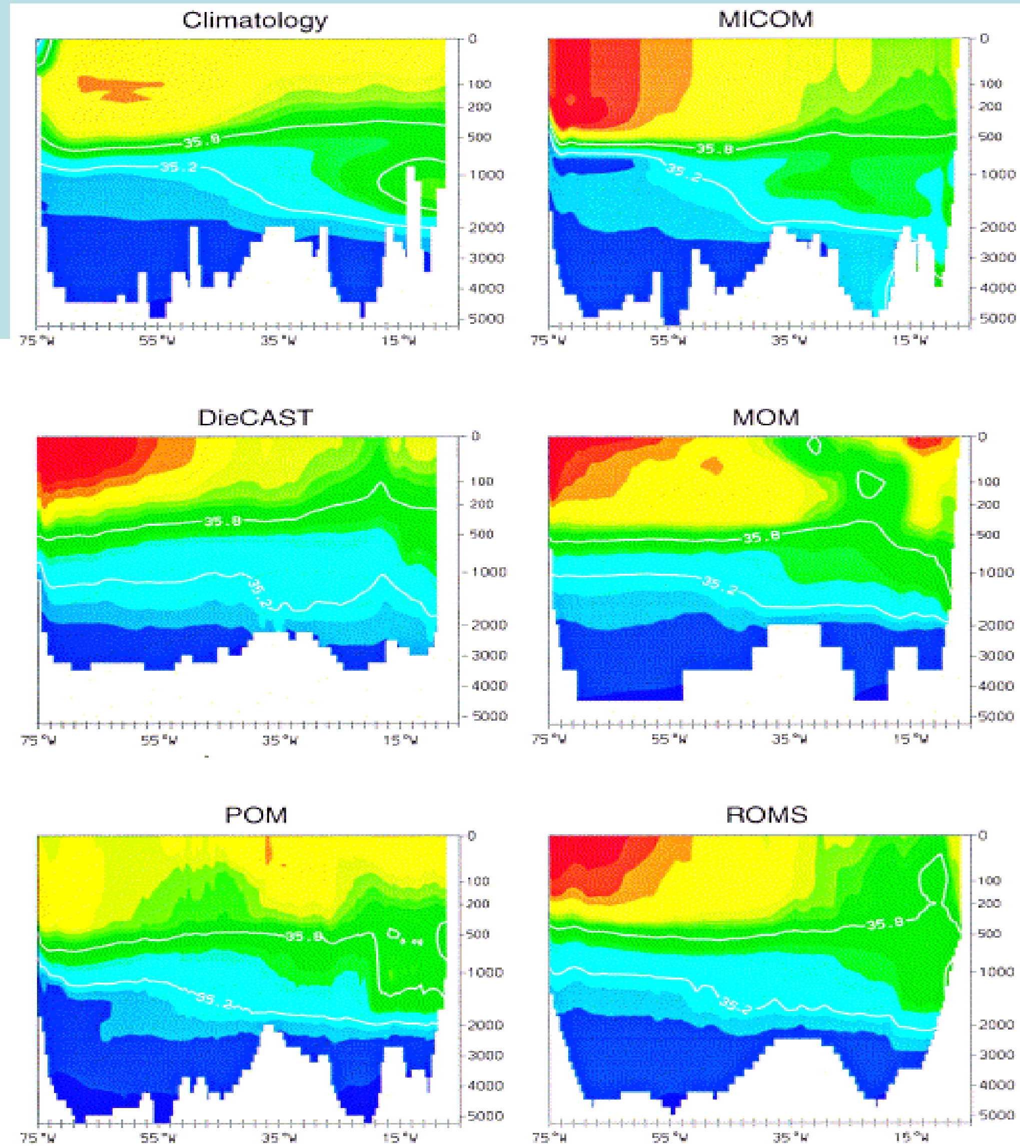


LEVEL

Salinity section at 35° N: Comparison with DAMÉE and A03

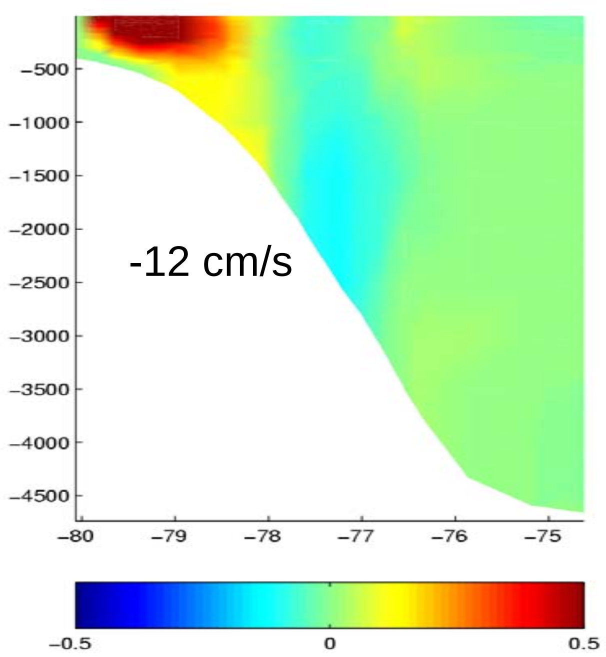






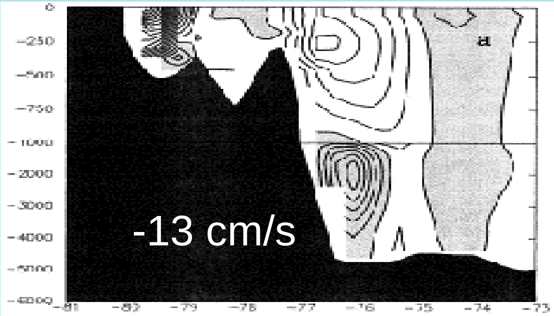
Western boundary currents at 27^{o} N

FEOM

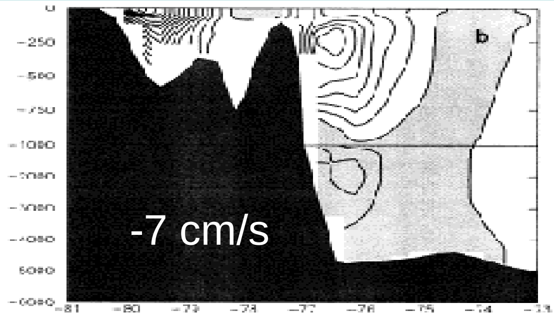


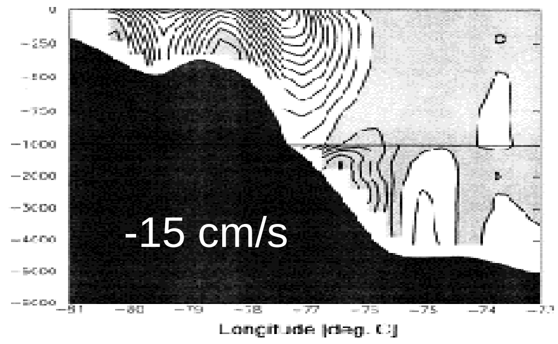
Core velocities are about 1 m/s for the Florida Current

LEVEL



ISOPYCNIC





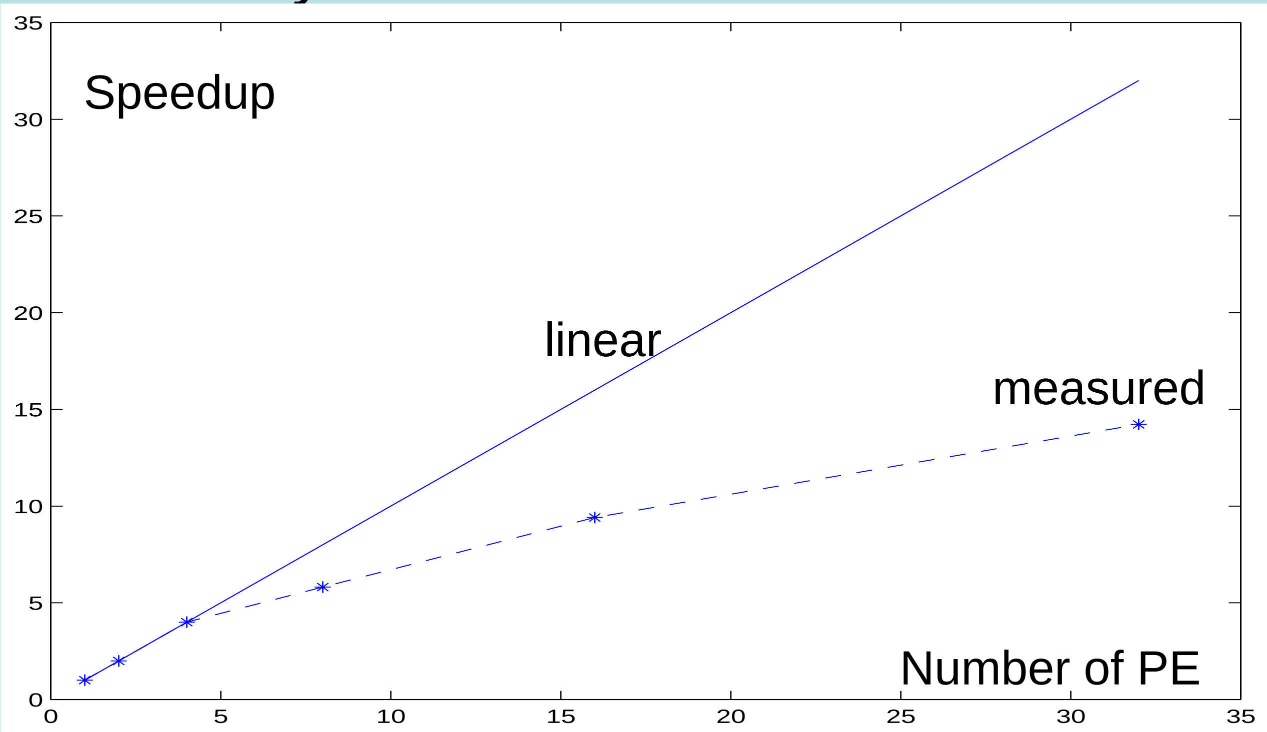
SIGMA

Transports:

	FC	DWBC
FEOM	37	17
LEVEL	35	17
SIGMA	36	16
ISOPYCNIC	38	11

Numerical cost and parallelization

MPI parallelization
Scalability:



Current cost 3.5 h per model year on 32 PE of IBM pSeries 690 (Hannover)

Vectorization --- future perspective. It requires sparse vectorized solvers and optimization of indirect addressing

Conclusions

FEOM is the first 3D FE primitive equation OGCM based on unstructured mesh

$\Downarrow$

(i) Variable resolution and smooth coastal line

(ii) Inclined bottom within z-coordinate

$\Downarrow$

Finite-elements could be used in climate ocean modelling